



Liquid–vapour fronts in porous media: Multiplicity and stability of front positions

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ABSTRACT

The mechanisms contributing to the stability and instability of the liquid–vapour front in a porous medium are analysed along a bifurcation curve with various combinations of thermal boundary conditions. In the first case, the boundaries above and below the layer of interest are assumed to be isothermal. It has been found that due to the competition between thermal and hydrostatic effects, the liquid–vapour interface may have multiple positions. A two-dimensional linear stability analysis of these basic states shows that the Rayleigh–Taylor mechanism is the dominant contributor to instability, but that there are circumstances under which the basic state may be stable. In the second case, a constant heat flux is imposed at the liquid boundary and a fixed temperature at the vapour boundary. It has been shown that the competition between the effects of cooling and the viscosity difference between the fluid phases causes multiple liquid–vapour front positions, whether or not gravity is considered. The stability analysis of such configurations has shown that along with the Rayleigh–Taylor (buoyancy-driven) mechanism, a Saffman–Taylor viscous fingering mechanism can also play an important role in the transition to instability.

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1. Introduction

There are many natural and industrial processes in which liquid and vapour phases exist in adjacent regions of a porous medium. Industrial contexts range from drying processes to soil remediation, while natural contexts include geothermal reservoirs and other geothermal systems [1–4].

A particularly interesting phenomenon is that, under certain circumstances, vertically stratified steady states can arise in which liquid overlies vapour (called ‘water-over-steam’ configurations in geothermal contexts). Despite the strong buoyancy contrast between the phases, these steady states may be stable. Their transition to instability has been shown [5–7] to have interesting mathematical properties: in particular, it may be ‘spontaneous’ in the sense that all wavenumbers become unstable simultaneously. A less thoroughly investigated feature of these configurations is that for particular boundary conditions multiple positions of the liquid–vapour front may be available. In this study we will demonstrate that this may occur not just for mixed thermal boundary conditions (as demonstrated by the seminal work of Rubin and Schweitzer [8]) but also for isothermal boundary conditions when a fixed pressure drop is imposed across the porous layer. Rubin and Schweitzer’s [8] work also suggested that the multiplicity and sta-

bility of front positions was directly linked; a second aim of our study is to demonstrate that this is not, in general, the case. Il’ichev and Tsytkin [9] showed that the multiple locations of the phase change front are permeability dependent and suggested that the middle front position is always unstable.

The complexity of liquid–vapour phase change problems means that there are useful roles both for comprehensive physical models of multiphase flow (for example, those that explicitly account for macroscale two-phase regions) and for more idealised models which elucidate the fundamental mathematical and physical aspects of the problems at the expense of some simplification. The present study belongs to the second category: we will assume that vapour and liquid each occupy a single-phase region, and make the simplest possible thermodynamical assumptions when deriving boundary conditions at the liquid–vapour front. This reduction appears to capture much of the essential physics while remaining mathematically tractable.

In Section 2 we will present the mathematical model we employ. We will then (Section 3) consider steady states under various boundary conditions on the system, focusing on the circumstances under which multiple front positions become available. In Section 4 we will present a linear stability analysis of these steady states, paying careful attention to the limiting behaviour of long- and short-wave perturbations. Finally, in Section 5 we will consider the physical interpretation and significance of our results.

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Nomenclature

Latin

\dot{m}	mass flux
c_p	specific heat
g	acceleration due to gravity
H	reciprocal of Stefan number
K	permeability
k	thermal conductivity
L	thickness of the low permeable layer
l	wave number
P	pressure
Pec	Peclet number
q	heat flux per unit area
S	location parameter of the interface
T	temperature
t	time
x	vertical coordinate
y	horizontal coordinate

Greek symbols

ϵ	perturbation parameter
κ	thermal conductivities ratio
λ	latent heat
μ	dynamic viscosity
ν	kinematic viscosity
ρ	density
σ	spectral parameter
σ^*	asymptotic spectral parameter
Θ	dimensionless temperature
Θ_0	temperature contrast ratio

v	fluid flow velocity
φ	porosity

Dimensionless quantities

C	specific heat ratio
E	heat capacity ratio
R	kinematic viscosity ratio
R_1	density ratio
R_2	dynamic viscosity ratio
R_3	Rayleigh number

Subscripts

L	liquid boundary
liq	liquid phase
m	porous medium
ref	reference quantity
S	at the phase transition front
s	porous skeleton
V	vapour boundary
vap	vapour phase
0	base state
1	perturbed state

Superscripts

min	minimum
$*$	dimensionless quantity
0	base state
1	perturbed state

2. Mathematical model

We consider a porous layer of infinite horizontal extension bounded above and below by two horizontal, much more permeable layers. The upper and lower highly permeable layers are filled either with vapour and liquid, respectively or liquid and vapour, respectively. In the low-permeability layer there exists a liquid–vapour front which separates the liquid phase from the vapour phase. The liquid side is kept cool, whereas the vapour side is hot. The highly permeable layers will allow us to impose constant pressures at both sides of the low-permeability layer.

2.1. Governing equations

Generally the differences in density between two phases (liquid and vapour) is much larger than the variations in density within a phase. Therefore, we will neglect changes of density within a phase and will take the density of each phase to be a (different) constant. The continuity equation for incompressible flow is

$$\nabla \cdot \vec{v}_i = 0, \text{ where } i = liq, vap \text{ and } \vec{v}_i = (u_i, v_i, w_i). \quad (1)$$

Darcy's equation is taken to hold in each phase,

$$\vec{v}_i = -\frac{K}{\mu_i} (\nabla P_i - \rho_i \vec{g}), \quad (2)$$

where $\vec{g} = -g \vec{e}_x$, K is the permeability of the homogeneous medium, μ is the dynamic viscosity, ρ is the density and g is the acceleration due to gravity. Regardless of the orientation of the layer, we take x to be zero on the liquid boundary and to increase towards the vapour boundary: thus if liquid overlies vapour then $g < 0$, and if vapour overlies liquid then $g > 0$.

We will use the one-equation model to describe the heat transport in the porous medium assuming local thermal equilibrium,

$$(\rho c)_{m,i} \frac{\partial T_i}{\partial t} + (\rho c_p)_i \vec{v}_i \cdot \nabla T_i = \nabla \cdot (k_{m,i} \nabla T_i). \quad (3)$$

where

$$(\rho c)_{m,i} = (1 - \varphi)(\rho c_p)_s + \varphi(\rho c_p)_i, \quad k_{m,i} = (1 - \varphi)k_s + \varphi k_i.$$

It can be seen from the above equation that the energy transport is coupled with the mass transport, which introduces non-linearities. Further non-linearities will be introduced by the coupling of the interface position with the heat and mass transport equations.

2.2. Interfacial mass and energy jump conditions

The contact between liquid and vapour means that the pressure and temperature at the front are related by $T = T_S(P)$ or $P = P_S(T)$, where T_S and P_S are phase change temperature and pressure, respectively. This relation introduces an inherited non-linearity associated with phase change problems [2, p. 320]. Il'ichev and Tsytkin [9] and Straus and Schubert [10] assumed that the temperature/pressure relationship determines the phase change condition. However, this assumption introduces a complicated empirical function, and it is not apparent that it is essential to the physics of instability. In the interest of simplicity, then, we will assume that a constant temperature and a continuous pressure at the liquid–vapour interface are prescribed,

$$\text{at } x = S: \quad P_{liq} = P_{vap}, \quad T_{vap} = T_{liq} = T_S, \quad (4)$$

where S is an arbitrary horizontal position of the liquid–vapour interface and T_S is the prescribed temperature at the interface. The thermodynamic properties (density, specific heat, viscosity, thermal conductivity, permeability) of each phase will be assumed constant but different.

The energy jump condition at the liquid–vapour interface ($x = S(y, z, t)$) is

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