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Numerical solution of a dynamic model of heat and moisture transfer in porous fabric under low temperature

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ABSTRACT

Dynamic heat and moisture transfer characteristics in human clothing directly determine human heat-moisture-comfort level. A dynamic coupled model of heat and moisture transfer with condensation in porous fabric under low temperature is fully considered and the coupled model together with variable initial and boundary conditions has been solved in theoretical way and numerical simulation. By decoupling of the dynamic model of heat and moisture transfer, the model is converted to a nonlinear parabolic partial differential equations, which is then numerically solved by an unconditionally stable implicit iterative scheme. The distributions of temperature, moisture concentration and water content in the porous fabric for different material in different environmental conditions are numerically computed and hence compared for different textile fibers. Numerical simulation results demonstrate validation of the mathematical model and effectiveness of the proposed numerical algorithms. It is expected to provide useful information for the functional material design based on heat and moisture transfer model and heat-moisture-comfort indexes.

1. Introduction

With the rapid development in science and technology, people have increasing requirements on uses of clothing for new functions, which contribute opportunities for further development and in corporation of new technologies and new materials in the recent years. In the aspect of the comfort textiles, we hope that the textiles are of fast decalescence or fast heat radiation. The microstructure and texture of textiles play a very important role in determining the heat-moisture comfort level of the human body. Hence people need take full account of the functional material design to improve the thermal protection function of textiles and garments.

Some scholars discussed a few dynamic models of heat and moisture transfer, and corresponding numerical simulations of thermal and water vapor concentration in porous media were given [1–12]. They took into account the physical characteristics (e.g. sorption, condensation in textiles), structure characteristics (e.g. porous media, multi-layered media) and various heat and mass transfer, momentum transfer characteristics (e.g. conduction, convection, radiation and molecular diffusion).

Henry [1] firstly proposed the establishment of coupled heat and moisture transfer model base on differential element, where two parabolic partial differential equations were presented to describe the process of heat and moisture transfer, meanwhile the coupling term was presented to describe the fiber absorption and desorption of moisture and the latent heat.

Until 1980s, there were some works on the coupling of heat and moisture transfer with phase change and condensation, which were carried out on the diverse aspects of simultaneous heat and moisture transfer in the literature both theoretically and experimentally. Under the assumptions that heat is transported by conduction and convection and the condensate is in pendular state, Ogniewicz and Tien [2] firstly investigated the analysis of the coupled heat and moisture transfer with condensation. Li and Holcombe [3] developed a new sorption rate equation containing two-stage sorption kinetics of wool fibers with more realistic boundary conditions to simulate the sorption behavior of wool fabrics.

In order to understand the thermal and moisture behavior of materials, numerical simulations is an effective way besides the experiment method. Some researchers, such as Fan and Li [4–12], have already put forward a lot of mathematical models of coupled heat and moisture transfer through porous clothing assemblies and fibrous insulation. Based on these models, they have presented different numerical methods to solve these problems, such as finite

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Nomenclature

C_a	water vapor concentration in the interfiber void space ($\mathrm{kg\,m^{-3}}$)
C_a^*	saturated water vapor concentration ($\mathrm{kg\,m^{-3}}$)
C_v	volumetric heat capacity of fabric ($\mathrm{kg\,m^{-3}\,k}$)
F_R	total thermal radiation incident on the element traveling to the right (W)
F_L	total thermal radiation incident on the element traveling to the left (W)
D_a	diffusion of water vapor in the air ($\mathrm{m^2\,s^{-1k}}$)
k	effective thermal conductivity of the fabric ($\mathrm{W\,m^{-1}\,K^{-1}}$)
L	thickness of the fabric (m)
RH	relative humidity (%)
T	temperature of the fabric (K)
x	distance (m)
w_0	water vapor resistance of the inner fabric
w_1	water vapor resistance of the outer fabric
W_c	sorption of moisture in the porous fibrous (%)
C_0	moisture concentrations surface next to human body at the boundary ($\mathrm{kg\,m^{-3}}$)
C_1	moisture concentrations surface next to surrounding air at the boundary ($\mathrm{kg\,m^{-3}}$)
h_c	convective vapor transfer coefficient between the outer surface of the outer fabric and the environment

Greek symbols

ε	porosity of the fabric
λ	latent heat of (de)sorption, vaporization or fusion ($\mathrm{kJ\,kg^{-1}}$)
ρ	density of the fabric ($\mathrm{kg\,m^{-3}}$)
τ	effective tortuosity of the fabric
β	absorption constant ($\mathrm{m^{-1}}$)
σ	boltzmann constant
Γ	sorption rate, condensation rate or freezing rate ($\mathrm{kg\,m^{-3}}$)

difference method, finite volume method, finite element method and control volume-time domain recursive method, and the numerical results are well matched with experimental results.

Based on the parallel pore structure textile and a body-clothing-environment system, Xu et al. [13] presented a model of heat and moisture transfer through the parallel pore textiles and numerically solved it by finite difference method. The proposed algorithm was proved convergent with the convergence rate of first-order theoretically. Xu continued to propose several inverse problems of textile thickness design for a single layer or bilayer textile material at low temperature based on ordinary differential equations in [14–16] or partial differential equations in [17]. Numerical results have provided insights into functional clothing design for cold Environment conditions. From present research results and the analysis of the basic features of various models, we can conclude that coupled models of heat and moisture transfer are the most promising theoretical model, so it is very necessary to mathematically study well-posed conditions and numerical algorithms for such problems.

In this paper, a dynamic model of heat and moisture transfer with sorption and condensation in porous fabric is fully considered in variable initial and boundary conditions. We reformulate the well-posed and complete dynamic coupled model with variable initial and boundary conditions in mathematical way and further solve the coupled model by the means of an unconditionally stable implicit iteration scheme. Under the result of the paper, we have proposed an inverse problem of textile thickness determination, which can predict and guide the textile design and clothing equipment design scientifically. The finding about the inverse problem of textile thickness determination has been reported in the journal of *Inverse Problems* [17].

The rest of the paper is composed of the following sections. In Section 2, we will introduce a mathematical model of a dynamic coupled heat and moisture transfer in porous fabric under a reasonable initial and boundary conditions. After decoupling the system of partial differential equations, we obtain two parabolic equations and our result corrects a formula in literature [8] in Section 3. In Section 4, the distributions of temperature, moisture concentration and water content in the porous media for different materials are numerically computed by the means of an unconditional stable implicit iteration scheme. Numerical simulation results indicate the validation of mathematical models and efficiency of numerical algorithms. We present some numerical results analysis for different materials and environmental conditions in Section 5. Comparing with other literature, we give several concluding remarks containing originality and improvements in the final section.

2. Numerical method

In this section, we consider a dynamic coupled model of heat and moisture transfer with condensation in porous fabric under low temperature [8]:

$$\begin{cases} C_v(x,t)\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k(x,t)\frac{\partial T}{\partial x}\right) + \frac{\partial F_L}{\partial x} - \frac{\partial F_R}{\partial x} + \lambda(x,t)\Gamma(x,t) \\ \frac{\partial F_L}{\partial x} = \beta F_L - \beta\sigma T^4(x,t) \\ \frac{\partial F_R}{\partial x} = -\beta F_R + \beta\sigma T^4(x,t) \\ \varepsilon\frac{\partial C_a}{\partial t} = \frac{D_a\varepsilon}{\tau}\frac{\partial^2 C_a}{\partial x^2} - \Gamma(x,t) \end{cases} \tag{1}$$

where $0 < x < L$, $0 < t < \mathbf{T}$. All kinds of heat and moisture processes, such as heat conductivity, radiation and sorption flow are considered in the single layer porous fabric batting in the above Eq. (1). We consider several dynamic functions including temperature, moisture concentration, (de)sorption, thermal radiation and water content in the heat and mass transfer process. In this work, we consider a model consisting of a thin inner fabric layer, a thick fibrous batting and a thin outer fabric, see Fig. 1. We adopt one assumption that the fibrous batting is isotropic.

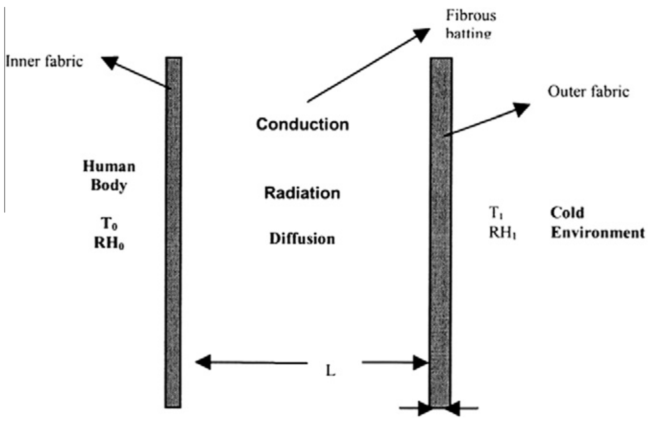

Fig. 1. Schematic diagram of the body-clothing-environment system consisting of a single layer fibrous batting.