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Natural convection flow and heat transfer in an eccentric annulus filled by Copper nanofluid

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ABSTRACT

A numerical study of natural convection flow and heat transfer of Copper (Cu)–water nanofluid inside an eccentric horizontal annulus is presented. The inner and outer cylinders are kept at constant temperatures as T_h and T_c, respectively. First the governing equations in terms of stream function-vorticity formulation in polar coordinate system for eccentric physical domain are derived and then transformed to a rectangular domain in order to get better accuracy of the solution near the boundaries. The resultant governing equations are discretized with a finite volume technique based on second order upwind scheme and then solved by iteration. The effects of the eccentricity ε, radii ratio (RR), the nanoparticles volume fraction parameter ϕ, the Rayleigh number Ra and the Prandtl number Pr on the mean Nusselt number Nu, streamlines and isotherms are investigated. The results are also discussed in detail. It is found that a very good agreement exist between the present results and those from the open literature.

1. Introduction

The convection heat transfer inside concentric and eccentric annulus have many applications in science and engineering, such as electrical motor and generator, completion of an oil source, heating and cooling of underground electric cables. Recently, investigation of the effect of eccentricity on heat transfer has become a subject of interest to most of researchers and they have studied the problem of convection heat transfer with various boundary conditions. It seems that the first person who worked on eccentric annuli was Heyda [1]. He applied a fundamental solution known as Green's function on solving the momentum equation for a laminar flow inside eccentric annulus. Then, Reynolds et al. [2], solved the problem of heat transfer for laminar and turbulent flow in eccentric annulus. They defined four types of boundary conditions that the temperature satisfied these boundary conditions. The temperature field has been introduced as a fundamental solution. Following Reynolds et al. [2], Trombetta [3] solved the energy equation of forced convection in an eccentric annulus assuming thermally and hydro-dynamically fully developed laminar flow using an approximate method. Bau [4] applied a perturbation series solution to solve natural convection heat transfer in a saturated porous medium confined between horizontal isothermal eccentric annulus. Both Trombetta [3] and Bau [4] indicated that there are values for eccentricity in which heat transfer inside annulus is optimized. They could also obtain a relation for the Nusselt number versus defined Rayleigh–Darcy number for a limited range of the Rayleigh–Darcy number. Himasekhar and Bau [5] solved the natural convection in eccentric annulus filled with saturated porous medium using the boundary layer technique. They could obtained a correlation for the Nusselt number as a function of the Rayleigh number, the radii ration and the eccentricity. Their correlations are valid for all Rayleigh numbers as long as the flow is steady. A numerical study was also made on natural convection in eccentric annuli with mixed boundary conditions by Ho et al. [6]. In this paper they indicated that the effect of Prandtl number on heat transfer is negligible although the heat and fluid patterns are affected by the Rayleigh number. They represent a correlation for Nusselt number versus the Rayleigh number. Hwang and Jensen [7] solved a simple form of the convective heat transfer equation for a thermally and hydro-dynamically fully developed laminar dispersed flow in eccentric annulus by the method of separation of variables. Mota et al. [8] presented a numerical simulation for natural convection in porous media confined between horizontal eccentric elliptic annuli. These authors claimed that stretching of one of the cylinders in the horizontal direction reduces the heat flow with respect to a concentric cylindrical annulus with the same radius ratio. Recently experimental studies on free and forced convection in an open-ended vertical eccentric annulus were carried out by Hosseini et al. [9,10]. They released that for turbulent flow ever the heat transfer rate increases with eccentricity while for laminar flow an optimum value for eccentricity is observed in which heat transfer is maximized.

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Nomenclature

e	dimensionless eccentricity, $e = \varepsilon/L$
g	gravity acceleration
K	thermal conductivity
L	characteristics length, $L = r_0 - r_i$
Nu	mean Nusselt number
Nu_L	local Nusselt number
$\bar{p}$	pressure field
Pr	Prandtl number
$\bar{r}_0$	radius of the outer cylinder
RR	radii ratio, $RR = r_0/r_i$
Ra	Rayleigh number
T	temperature of the nanofluid
T_c	temperature of the outer cylinder
T_h	temperature of the inner cylinder
$\bar{u}$	radial velocity
$\bar{v}$	tangential velocity
X	radial distance between concentric cylinders
Greek symbols	
α	thermal diffusivity
ε	eccentricity
ϕ	nanoparticle volume fraction
η	dimensionless radial coordinate
ν	kinematic viscosity
θ	angular coordinate
Θ	dimensionless temperature
ρ	density
ξ	dimensionless angular coordinate
Subscripts	
c	cold
h	hot
i	inner cylinder
o	outer cylinder
f	base fluid
nf	nanofluid
s	solid nanoparticles

Further, they found that the eccentric annulus conclude greater heat transfer coefficient in comparison with concentric annulus. Nobari and Mehrabani [11] conducted a numerical analysis in forced convection in eccentric curved annuli for various boundary conditions. They concluded that for the straight eccentric annuli comparing with eccentric annuli, the heat transfer rate can be augmented in the eccentric curved annuli at the large Dean numbers depending on the eccentricity and the curvature ratio.

In recent years, convective heat transfer from nanofluids has been noticeable. Conventional fluids, such as water, ethylene glycol mixture and some types of oil have low heat transfer coefficient, the reason for which might be related to the low conduction coefficient of these fluids. In most heat transfer studies, the base fluid has a low thermal conductivity, which limits the heat transfer enhancement. However, the continuing miniaturization of electronic devices requires further heat transfer improvements from an energy saving viewpoint. An innovative technique, which uses a mixture of nanoparticles and the base fluid was first introduced by Choi [12] in order to develop advanced heat transfer fluids with substantially higher conductivities. The resulting mixture of the base fluid and nanoparticles having unique physical and chemical properties is referred to as a nanofluid. It is expected that the presence of the nanoparticles in the nanofluid increases the thermal conductivity and therefore substantially enhances the heat transfer characteristics of the nanofluid. Eastman et al. [13] and Xie et al. [14] showed that higher thermal conductivity can be achieved in thermal systems using nanofluids. Choi et al. [15] affirmed that the addition of a one percent by volume of nanoparticles to usual fluids increases the thermal conductivity of the fluid up to approximately two times. As regards the convective flow of nanofluids in annulus it seems that Soleimani et al. [16] are the first who have presented a numerical finite element solution for natural convection nanofluid flow in a semi-annulus enclosure. Furthe, Abu-Nada [17] and Abu-Nada et al. [18] investigated natural convection of nanofluid in a concentric annulus considering variable viscosity and variable thermal conductivity. The mathematical nanofluid model proposed by Khanafer et al. [19] or Tiwari and Das [20] has been used. It is worth mentioning at this end that Putra et al. [21] have presented an experimental study of natural convection of nanofluids inside a horizontal cylinder heated from one end and cooled from the other.

Many researchers have investigated the effects of nanofluids on the enhancement of heat transfer in thermal engineering devices, both experimentally and theoretically, and the number of publications related to nanofluids increases in an exponential manner during the last several years. Detailed descriptions of nanofluids can be found in the book by Das et al. [22], and in the review papers by Buongiorno [23], Ding et al. [24], Wang and Mujumdar [25], Choi [26], Kakaç and Pramuanjaroenkij [27], Li et al. [28], Lee et al. [29], Wong and Leon [30], Yu and Lin [31], Ghadimi et al. [32], Ramesh and Prabhu [33], Sarkar [34], Fan and Wang [35], Saidur et al. [36], and Thomas and Sobhan [37].

Although the nanofluid flow in some of geometries is studied in literature so far the effect of nanofluid in natural convection heat transfer inside the eccentric annulus as a complex geometry has not been investigated. In this paper a laminar natural convection flow and heat transfer in a horizontal two-dimensional eccentric annulus filled with Copper (Cu)–water nanofluid has been studied numerically. The horizontal cylinders are kept at constant temperatures as the outer cylinder is cold and the inner cylinder is hot. The governing partial differential equations are transformed to rectangular coordinates with a transformation, which enables the equations to be discretized on an orthogonal uniform mesh. The obtained equations are discretized using finite volume technique based on second order upwind scheme and then the resultant algebraic equations are solved by iteration.

2. Mathematical formulation

The steady-state two dimensional laminar natural convection of a nanofluid in an annuli of eccentric horizontal cylinders of radii r_i and r_o is considered. The outer cylinder is cooled at the temperature T_c and the inner cylinder is kept at the hot temperature T_h. The distance between the cylinders' center, the eccentricity e, is denoted as positive when the center of the inner cylinder is above the center of the outer cylinder. Assuming that the properties of the nanofluid are constant except that its density is variable and satisfies the Boussinesq approximation, the governing dimensional equations in terms of the stream function-vorticity formulation are the kinematic, the energy and vorticity equations, which can be written in cylindrical coordinates as