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Transition to chaos in thermocapillary convection

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ABSTRACT

We present experiments on transition to chaos in thermocapillary convection in a rectangular pool of silicone oil. The applied temperature difference between the two sidewalls is adjusted in the range of 0–43 °C to observe various dynamic states. The applied temperature gradient along the fluid–gas interface drives shear flow along the free surface from hot to cold and a back flow in the underlying layer. With the increase of the temperature gradient, the thermocapillary convection will transit from steady flow to regularly oscillatory flow, and finally to chaos. A temperature measurement system, which consists of thermocouple, voltmeter and data-acquiring computer, is used to record the temperature of the liquid dynamically. In order to identify the different dynamic regimes from steady flow to chaos, fast Fourier transform and fractal theory are used to analyze the experimental data. The critical conditions for transition have been obtained and discussed by non-dimensional analysis. The quasi-periodic route and Feigenbaum route were observed for different experimental conditions, and the relationship between oscillatory frequency and Marangoni number *Ma* has been discussed.

1. Introduction

Thermocapillary convection is driven through the surface tension gradient, which is produced by a temperature gradient along the free surface. When the applied temperature gradient is increased, the convection will transit from steady flow to a sequence of instabilities, and finally to chaos or turbulence. The transition to chaos in thermocapillary convection has been of interest for its practical and theoretical value [1,2]. Critical conditions and transition routes are two important aspects of transition to chaos.

A large amount of researches has been done on the critical conditions and instabilities at threshold. Smith and Davis [3,4] performed a linear stability analysis of thermocapillary instability. When the free surface is assumed to be flat and non-deformable, they found two types of thermocapillary instabilities: stationary longitudinal rolls and hydrothermal waves. When the free surface is thought to be deformable, the instability of surface wave was obtained. And then the theoretical analysis of the thermocapillary instabilities have been made up by Parmentier [5] for the consideration of buoyant effect, Mercier and Normand [6] for the introduction of heat exchange to the atmosphere and Kuhlmann [7] for three-dimensional flow. Experiments on thermocapillary instability have been conducted by Riley and Neitzel [8]. They have observed two kinds of instabilities through instantaneous thermograph in a rectangular pool. For small Bond number, the original steady unicellular flow transits to hydrothermal waves. For large Bond number, it transits to steady multicellular flow first, and then to oscillating multicellular flow, which features steady multicellular structures near the hot wall and a pair of oblique waves near the cold wall. Not only the hydrothermal waves, but Burguete [9] has also observed the instability of stationary rolls for deep liquid layer by means of shadowgraph images. Critical temperature differences for several aspect ratios have been collected.

Transition routes from laminar to turbulence regime in Rayleigh–Benard convection [10–13] and Benard–Marangoni convection [14] have been studied extensively both experimentally and numerically. While the temperature-gradient directions in the convections mentioned above are in vertical, transition routes in thermocapillary convection applied with horizontal temperature-gradient have been much less investigated. Bucchignani and Mansutti [15,16] reported the numerical results of bifurcation pattern of thermocapillary convection. They used the Rayleigh number *Ra* as the bifurcation parameter. When *Ra* = 4.25e8, an unsteady periodic flow with a fundamental frequency was obtained. Then an increase of *Ra* at 4.3e8 leads to the second Hopf bifurcation from the periodic flow to a quasi-periodic regime with two incommensurate frequencies. At *Ra* = 5e8, the presence of a quasi-periodic regime with three incommensurate frequencies has been observed. And then it develops into chaotic flow. This kind of bifurcation sequence with the characteristic of quasi-periodic bifurcation is named Ruelle–Takens–Newhouse route, which is one of three well-known routes to chaos. [2] Besides the Ruelle–Takens–Newhouse route, the Feigenbaum

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Nomenclature

h	depth of the liquid layer, m
t	time, s
v	dynamic viscosity, Pa s
γ	kinetic viscosity, $m^2\ s^{-1}$
L_x	stream-wise domain length, m
g	gravitational acceleration, $m\ s^{-2}$
T	temperature of the measurement point, °C
$\triangle T$	applied temperature difference between the two sidewalls, °C
f_1	the first fundamental frequency, Hz
f_2	the second fundamental frequency, Hz

Greek symbols

β	thermal expansion coefficient, 1/°C
κ	thermal diffusivity, $m^2\ s^{-1}$
ρ	density, $kg\ m^{-3}$
σ	surface tension, $N\ m^{-1}$

route is characterized by period-doubling bifurcation, and the Pomeau–Manneville route by intermittent appearance of periodic and chaotic phases. The other two transition routes were not reported in Mansutti's study. There is also a lack of experimental results of transition routes in thermocapillary convection in rectangular pool.

Study on transition to chaos in thermocapillary convection is beneficial to explore the mechanism of thermocapillary instability. And experiments on transition routes of thermocapillary convection in rectangular pool need to be studied to make up the lack. The aim of the present research work is to obtain critical conditions for the transition, and to identify transition routes to chaos. To achieve the goal, we designed a temperature measurement system with high resolution to record the temperature of the liquid dynamically. In order to identify the transition routes from steady flow to chaos, fast Fourier transform and fractal theory are used to analyze the experimental data.

The paper is organized as follows: Section 2 gives a brief description of the thermocapillary problem and the analysis method to investigate it. In Section 3, we first study the critical conditions for the transition; our results are discussed and compared with others. Then, transition routes to chaos are studied for two experimental conditions; transitions in the two conditions follow the quasi-periodic route and Feigenbaum route, which is identified experimentally in thermocapillary convection in rectangular pool for the first time. Finally, the main findings are summarized in Section 4.

2. Experimental setup and procedure

In order to research on evolvement of temperature oscillation in our laboratory, we have constructed a buoyancy–thermocapillary convection system as shown in Fig. 1. The rectangle pool is made up of a right hot end with the thickness of 6 mm made of copper heated by an electrothermal film and a left cold end with the same thickness. The horizontal cross-section of the pool is 52 × 36 mm, and the height of the pool is 6 mm. The front, rear and bottom side of the pool is made of optical glass K9 with the thickness of 6 mm. In our experiment, the working fluid is silicone oil of 0.65, 1, and 1.5 cSt whose Prandtl numbers are equal to 10, 16, and 25 respectively. The horizontal temperature gradient in the fluid layer will be established between the two copper walls. A DC electrical power is controlled by a temperature controller to work the electrothermal film to heat the hot end. The temperatures of the two copper walls are measured through two T-type thermocouples. With the increase of the temperature difference between the two copper walls, the convection in the fluid layer in the rectangle pool will transit from stable to unstable. The working fluid used in our experiments is silicone oil 0.65, 1, and 1.5 cSt, whose physical properties are given in Table 1.

To record the temperature evolvement in the fluid layer in a period of time, we have designed a temperature measurement system, which consists of a thermocouple, a voltmeter, and a personal computer. The sensor in the system is thermocouple, the diameter of whose wire is about 60 μm. The operating principle is shown in Fig. 2. Thermocouple transfers the temperature signal to voltage signal, which will be measured by the voltmeter. Finally, the temperature is calculated and is recorded by personal computer. During our experiment, the temperature magnitudes are recorded with the sampling rate of 5 Hz.

To identify the different dynamic regimes during the transition to chaos, the fast Fourier transform is used to calculate the power spectrum of a dynamic variable. Periodic, quasi-periodic and period-doubling flows can be recognized from the power spectra. Chaotic flow is considered to occur, when a signal broadband develops in the power spectrum.

Fractal theory becomes more and more popular in the application of chaotic dynamic analysis. Chaotic time series can be represented with a fractal through reconstruction technique, which stems from the embedding theorem developed by Takens [17] and Sauer [18]. Fractal dimension D, one of the important and internal characteristics of a fractal, is usually estimated by calculating correlation dimension D_m. GP algorithm proposed by Grassberger and Procaccis [19] is a simple and reliable method to determine D_m. In the method, a correlation integral $C_m(r)$ is defined as

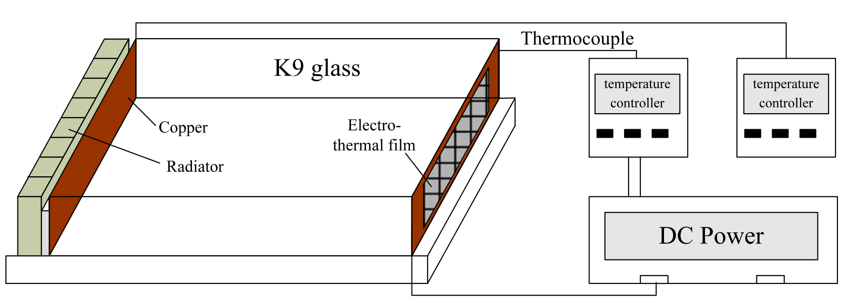

Fig. 1. Controlling system for buoyancy-thermocappilary convection.