International Journal of Heat and Mass Transfer 57 (2013) 465–472

Contents lists available at SciVerse ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Homogeneous–heterogeneous reactions in a nanofluid flow due to a porous stretching sheet

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ARTICLE INFO

Article history:
Received 24 August 2012
Received in revised form 13 October 2012
Accepted 14 October 2012
Available online 23 November 2012

Keywords:
Homogeneous–heterogeneous reactions
Nanofluids
Volume fraction model
Stretching sheet

ABSTRACT

We investigate the effects of homogeneous–heterogeneous reactions in nanofluid flow over a stretching or shrinking sheet placed in a porous medium saturated with a nanofluid. Copper–water and silver–water nanofluids are investigated in this study. The steady states of this system are analyzed in the case when the diffusion coefficients of the reactant and auto catalyst are equal. The governing partial differential equations are transformed into a system of non-linear ordinary differential equations and solved numerically. An analytical solution for the momentum equation is obtained.

1. Introduction

The study of convective flow through porous media has received a great deal of research interest over the last three decades due to its wide and important applications in environmental, geophysical and industrial problems. Prominent applications include the utilization of geothermal energy, the migration of moisture in fibrous insulation, drying of a porous solid, food processing, casting and welding in manufacturing processes, the dispersion of chemical contaminants in different industrial processes, the design of nuclear reactors, chemical catalytic reactors, compact heat exchangers, solar power and many others.

The problem of a stretching surface has many engineering applications such as in extrusion processes, melt-spinning, hot rolling, wire drawing, glass-fiber production, manufacture of plastic and rubber sheets. Other applications can be found in the manufacture of polymer sheets, food processing and in the movement of biological fluids, [1–4]. Crane [5] was the first to consider steady two-dimensional flow of a Newtonian fluid driven by a stretching elastic flat sheet which moves in its own plane with a velocity varying linearly with the distance from a fixed point. This was subsequently extended by many authors to explore various aspects of heat transfer in a fluid surrounding a stretching sheet [6–15].

Abel et al. [16] investigated the effect of porous medium on the flow and heat transfer of a nonuniform viscoelastic liquid over a non-isothermal stretching sheet. Abel et al. [17] presented a mathematical analysis for the momentum and heat transfer characteristics of the boundary layer flow of an electrically conducting viscoelastic fluid over a linearly stretching sheet. Pal and Mondal [18] studied the influence of a chemical reaction and thermal radiation on a stretching sheet in a Darcian porous medium with Soret and Dufour effects. They found that the temperature profiles increase as the thermal radiation parameter and the Dufour number increase. Recently, Rosali et al. [19] investigated micropolar fluid flow over a stretching/shrinking sheet in a porous medium with suction. They observed that an increase in the permeability parameter led to an increase in the skin friction coefficient and the local Nusselt number.

Many chemically reacting systems involve both homogeneous and heterogeneous reactions, with examples occurring in combustion, catalysis and biochemical systems. The interaction between the homogeneous reactions in the bulk of the fluid and heterogeneous reactions occurring on some catalytic surfaces is generally very complex, and is involved in the production and consumption of reactant species at different rates both within the fluid and on the catalytic surfaces. A model for isothermal homogeneous–heterogeneous reactions in boundary layer flow of a viscous fluid past a flat plate was studied by Merkin [20]. He presented the homogeneous reaction by cubic autocatalysis and the heterogeneous reaction by a first order process and showed that the surface reaction is the dominant mechanism near the leading edge of the plate. Chaudhary and Merkin [21] studied homogeneous–heterogeneous reactions in boundary layer flow. They studied the numerical solution near the leading edge of a flat plate. Ziabakhsh et al. [22] studied the problem of flow and diffusion of chemically

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http://dx.doi.org/10.1016/j.ijheatmasstransfer.2012.10.047

reactive species over a nonlinearly stretching sheet immersed in a porous medium. Chambre and Acrivos [23] studied an isothermal chemical reaction on a catalytic reactor in laminar boundary layer flow. They found the actual surface concentration without introducing unnecessary assumptions related to the reaction mechanism. The effects of flow near the two dimensional stagnation point flow on an infinite permeable wall with a homogeneous–heterogeneous reaction was studied by Khan and Pop [24]. They solved the governing nonlinear equations using the implicit finite difference method and observed that the mass transfer parameter considerably affects the flow characteristics. Khan and Pop [25] studied the effects of homogeneous–heterogeneous reactions on a viscoelastic fluid toward a stretching sheet. They observed that the concentration at the surface decreased with an increase in the viscoelastic parameter.

Convectional heat transfer fluids, including oil, water, and ethylene glycol mixtures are poor heat transfer fluids due to the low thermal conductivity of these fluids. The thermal conductivity of these fluids may be improved by suspending nano sized particle materials in the liquid to form a nanofluid. A characteristic feature of nanofluids is thermal conductivity enhancement, a phenomenon observed by Masuda et al. [26]. Thermophysical properties of nanofluids such as thermal conductivity, diffusivity and viscosity have been studied by, among others, Kang et al. [27], Velagapudi et al. [28] and Rudyak et al. [29]. Boundary layer flow of a nanofluid past a stretching sheet with a convective boundary condition was studied by Makinde and Aziz [30]. They found that with an increase in the Biot number, the concentration layer thickened, but the concentration layer became thinner as the Lewis number increased. Alsaedi et al. [31] studied the effects of heat generation/absorption on stagnation point flow of nanofluid over a surface with convective boundary conditions. Recently, Narayana and Sibanda [32] studied the effects of laminar flow of a nanoliquid film over an unsteady stretching sheet. They found that the effect of the nanoparticle volume fraction is to reduce the axial velocity and free stream velocity in the case of a Cu-water nanoliquid, but the opposite is true in the case of a Al_2O_3 – water nanoliquid. Kameswaran et al. [33] studied hydromagnetic nanofluid flow due to a stretching or shrinking sheet with viscous dissipation and chemical reaction effects. They found that the velocity profile decreased with an increase in the nano particle volume fraction, while the opposite was true in the case of temperature and concentration profiles. The study further showed that liquids with nanoparticle suspensions are better suited for effective cooling of the stretching sheet problem due to their enhanced conductivity and thermal properties.

In this paper we study the combined effects of a porous medium permeability parameter subject to homogeneous–heterogeneous reactions in a nanofluid flow over a stretching or shrinking sheet. The model developed by Merkin [20] for homogeneous–heterogeneous reactions in boundary layer flow with cubic autocatalysis is used in the present study. The transformed ordinary boundary layer equations of motion and concentration are solved numerically. To the best of the authors' knowledge, this problem has not been considered before in the literature.

2. Mathematical formulation

Consider two dimensional steady boundary layer flow of an incompressible nanofluid over a stretching sheet. A cartesian coordinate system is used with the x-axis along the sheet and the y-axis normal to the sheet. Two equal but opposite forces are applied along the sheet so that the wall is stretched, keeping the position of the origin unaltered. The fluid is a water based nanofluid containing copper (Cu) or silver (Ag) nanoparticles. The base fluid and the nanoparticles are in thermal equilibrium and no slip occurs between them. It is assumed that a simple homogeneous–heterogeneous reaction model exists as proposed by Chaudhary and Merkin [21] in the following form:

$$A + 2B \rightarrow 3B, \quad \text{rate} = k_c a b^2, \tag{1}$$

while on the catalyst surface we have the single, isothermal, first order reaction

$$A \rightarrow B, \quad \text{rate} = k_s a, \tag{2}$$

where a and b are the concentrations of the chemical species A and B, k_c and k_s are the rate constants. We assume that both reaction processes are isothermal. Under these assumptions, the boundary layer equations governing the flow can be written in dimensional form [21,34];

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{3}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u}{\partial y^2} - \frac{\mu_{nf}}{\rho_{nf}}\frac{1}{k}u, \tag{4}$$

$$u\frac{\partial a}{\partial x} + v\frac{\partial a}{\partial y} = D_A\frac{\partial^2 a}{\partial y^2} - k_c a b^2, \tag{5}$$

$$u\frac{\partial b}{\partial x} + v\frac{\partial b}{\partial y} = D_B\frac{\partial^2 b}{\partial y^2} + k_c a b^2. \tag{6}$$

The boundary conditions for Eqs. (3)–(6) are given in the form:

$$\begin{aligned} &u = u_w = cx, \quad v = 0, \quad D_A\frac{\partial a}{\partial y} = k_s a, \quad D_B\frac{\partial b}{\partial y} = -k_s a \quad \text{at} \quad y = 0, \\ &u \rightarrow 0, \quad a \rightarrow a_0, \quad b \rightarrow 0 \quad \text{as} \quad y \rightarrow \infty, \end{aligned} \tag{7}$$

where u, v are the velocity components in the x-and y-directions respectively, c is the stretching ($c > 0$) or shrinking ($c < 0$) rate, k is the permeability of the porous medium, D_A and D_B are the respective diffusion species coefficients of A and B, a_0 is a positive constant. The effective dynamic viscosity of the nanofluid was given by Brinkman [35] as

$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}, \tag{8}$$

where ϕ is the solid volume fraction of nanoparticles. The effective density of the nanofluids is given as

$$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_s. \tag{9}$$

The thermal diffusivity of the nanofluid is

$$\alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}}, \tag{10}$$

where the heat capacitance of the nanofluid is given by

$$(\rho C_p)_{nf} = (1-\phi)(\rho C_p)_f + \phi(\rho C_p)_s. \tag{11}$$

Here, the subscripts nf, f and s represents the thermophysical properties of the nano fluid, base fluid and nano solid particles, respectively. The continuity Eq. (3) is satisfied by introducing a stream function $\psi(x,y)$ such that

$$u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x},$$

where $\psi = (c\nu_f)^{\frac{1}{2}} x f(\eta)$, $f(\eta)$ is the dimensionless stream function and $\eta = (c/\nu_f)^{\frac{1}{2}} y$.

The velocity components are then given by

$$u = cxf'(\eta) \quad \text{and} \quad v = -(c\nu_f)^{\frac{1}{2}} f(\eta). \tag{12}$$

The concentrations of the chemical species A and B are represented as