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Non-orthogonal stagnation point flow of a nano non-Newtonian fluid towards a stretching surface with heat transfer

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ABSTRACT

This article investigates the theoretical study of steady stagnation point flow with heat transfer of a second grade nano fluid towards a stretching surface. It is assumed that the fluid impinges on the wall obliquely. The model used for the nano fluid incorporates the effects of Brownian motion and thermophoresis. The governing equations of second grade nano fluid are presented. The governing nonlinear partial differential equations are converted into nonlinear ordinary differential equations by using similar and non similar variables. The resulting ordinary differential equations are successfully solved analytically with the help of homotopy analysis method (HAM). Graphically results are shown for nondimensional velocities, temperature and nanoparticle concentration. Numerical values of skin friction coefficients, diffusion mass flux and heat flux are computed. It is shown that a boundary layer is formed when the stretching velocity of the surface is less than the inviscid free-stream velocity and velocity at a point increases with the increase in the elasticity of the fluid. Comparison with previously published work is performed and excellent agreement is observed for the limited case of existing literature.

1. Introduction

Nanofluid is a fluid containing nanometer-sized particles, called nanoparticles. These fluids are engineered colloidal suspensions of nanoparticles in a base fluid [1]. The nanoparticles used in nanofluids are typically made of metals, oxides, carbides, or carbon nanotubes. Common base fluids include water, ethylene glycol and oil. Nanofluids have novel properties that make them potentially useful in many applications in heat transfer, including microelectronics, fuel cells, pharmaceutical processes, and hybrid-powered engines [2] engine cooling/vehicle thermal management, domestic refrigerator, chiller, heat exchanger, nuclear reactor coolant, in grinding, machining, in space technology, defense and ships, and in boiler flue gas temperature reduction. A very small amount of relatively higher thermal conductivity nanometer-sized particles (less than 100 nm), when dispersed uniformly and suspended stably in conventional fluids, can provide dramatic improvements in the thermophysical properties of the base fluids. They exhibit enhanced thermal conductivity and the convective heat transfer coefficient compared to the base fluid. In analysis such as computational fluid dynamics, nanofluids can be assumed to be single phase fluids. Classical theory of single phase fluids can be applied, where physical properties of nanofluid is taken as a function of properties of both constituents and their concentrations [3]. The problem of laminar fluid flow which results from the stretching of a flat surface in a nanofluid has been investigated numerically by Khan and Pop [4]. They found that the reduced Nusselt number is a decreasing function of each dimensionless number, while the reduced Sherwood number is an increasing function of higher Prandtl number and a decreasing function of lower Prandtl number for each Lewis, Brownian motion and thermophoresis numbers. The influence of nanoparticles on natural convection boundary-layer flow past a vertical plate have been examined by Kuznetsov and Nield [5]. The authors have assumed the simplest possible boundary conditions in which both the temperature and the nanoparticle fraction are constant along the wall. The steady boundary layer free convection flow past a horizontal flat plate embedded in a porous medium filled by a water-based nanofluid containing gyrotactic microorganisms is investigated by Aziz et al. [6]. Vajravelu et al. [7] discussed the convective heat transfer in the flow of viscous Ag-water and Cu-water nanofluids over a stretching surface. According to their analysis nanoparticles results in an increase in the magnitude of the skin friction along the surface and a decrease in the magnitude of the local Nusselt number. Transport phenomena of viscoelastic nanofluid over a stretching sheet with energy dissipation has been studied by Rana and Bhargava [8]. They found that Heat transfer is increasing function of viscoelastic parameter and decreasing function of Brownian motion, thermophoresis and Eckert number. Noghrehabadi et al. [9] analyzed the development of the slip effects

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Nomenclature

C	nanoparticle concentration
T_w	temperature at the stretching surface
T_∞	ambient temperature as y tends to infinity
C_w	nanoparticle concentration at stretching surface
C_∞	ambient nanoparticle concentration as y tends to infinity
c_p	the specific heat of the material
D_B	Brownian diffusion coefficient
D_T	thermophoresis diffusion coefficient
Le	Lewis number
N_b	Brownian motion parameter
N_t	thermophoresis parameter
Pr	Prandtl number
We	Weissenberg number
P	fluid pressure
T	fluid temperature
u, v	velocity components along x and y axes respectively
x, y	Cartesian coordinates measured along stretching surface
ν	kinematic viscosities
$(\rho c)_p$	effective heat capacity of the nanofluid
$(\rho c)_f$	heat capacity of the fluid
α	thermal diffusivity
Ψ	stream function
$f'(y)$	dimensionless velocity
$\theta(y)$	dimensionless temperature
$\phi(y)$	dimensionless concentration

on the boundary layer flow and heat transfer over a stretching surface in the presence of nanoparticle fractions. They observed that the flow velocity and the surface shear stress on stretching sheet are strongly influenced by the slip parameter and by the increase in velocity slip factor (λ) the momentum boundary layer thickness and thermal boundary layer thickness are decreased and increased respectively. Some interesting recent investigations related to the topic are presented in Refs. [10–16]. The present study looks for the non-orthogonal stagnation point flow of a nano second grade fluid. The structure of the paper is organized as follow. The flow problem is formulated in section two. Section three and four deal with the series solution and their convergence respectively. Results and discussions are given in section five. Section six consists of conclusions. At the end, the physical behavior of pertinent parameters have been discussed.

2. Mathematical formulation

We investigate the steady two-dimensional stagnation point flow of a second grade nano fluid over a stretching surface. Two equal and opposite forces are applied along the x-axis so that the surface is stretched keeping the origin fixed, as shown in Fig. 1a. We further assume that the surface has temperature T_w and fluid has uniform ambient temperature T_∞ (Here $T_w > T_\infty$).

The governing equations of second grade nano fluid are defined as [1]

$$\frac{\partial u^*}{\partial x^*}+\frac{\partial v^*}{\partial y^*}=0, \tag{1}$$

$$\begin{aligned}
&u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}+\frac{1}{\rho_f}\frac{\partial p^*}{\partial x^*}\\
&=(\nu)\nabla^{*^2}u^*+\frac{\alpha_1}{\rho_f}\left\{\frac{\partial}{\partial x^*}\left[4\left(\frac{\partial u^*}{\partial x^*}\right)^2+2\frac{\partial v^*}{\partial x^*}\left(\frac{\partial u^*}{\partial y^*}+\frac{\partial v^*}{\partial x^*}\right)+2u^*\left(\frac{\partial^2 u^*}{\partial x*^2}\right)+2v^*\left(\frac{\partial^2 u^*}{\partial y*\partial x*}\right)\right]\right.\\
&\left.+\frac{\partial}{\partial y^*}\left[2\left(\frac{\partial u^*}{\partial x^*}\right)\left(\frac{\partial u^*}{\partial y^*}\right)+2\left(\frac{\partial v^*}{\partial x^*}\right)\left(\frac{\partial v^*}{\partial y^*}\right)+\left(u^*\frac{\partial}{\partial x^*}+v^*\frac{\partial}{\partial y^*}\right)\left(\frac{\partial u^*}{\partial y^*}+\frac{\partial v^*}{\partial x^*}\right)\right]\right\}\\
&+\frac{\alpha_2}{\rho_f}\frac{\partial}{\partial x^*}\left[4\left(\frac{\partial u^*}{\partial x^*}\right)^2+\left(\frac{\partial u^*}{\partial y^*}+\frac{\partial v^*}{\partial x^*}\right)^2\right],
\end{aligned}\tag{2}$$

$$\begin{aligned}
&u^*\frac{\partial v^*}{\partial x^*}+v^*\frac{\partial v^*}{\partial y^*}+\frac{1}{\rho_f}\frac{\partial p^*}{\partial y^*}\\
&=(\nu)\nabla^{*^2}v^*+\frac{\alpha_1}{\rho_f}\left\{\frac{\partial}{\partial x^*}\left[2\left(\frac{\partial u^*}{\partial x^*}\right)\left(\frac{\partial u^*}{\partial y^*}\right)+2\left(\frac{\partial v^*}{\partial x^*}\right)\left(\frac{\partial v^*}{\partial y^*}\right)+\left(u^*\frac{\partial}{\partial x^*}+v^*\frac{\partial}{\partial y^*}\right)\left(\frac{\partial u^*}{\partial y^*}+\frac{\partial v^*}{\partial x^*}\right)\right]\right.\\
&\left.+\frac{\partial}{\partial y^*}\left[4\left(\frac{\partial v^*}{\partial y^*}\right)^2+2\frac{\partial u^*}{\partial y^*}\left(\frac{\partial u^*}{\partial y^*}+\frac{\partial v^*}{\partial x^*}\right)+2v^*\left(\frac{\partial^2 v^*}{\partial x*^2}\right)+2u^*\left(\frac{\partial^2 v^*}{\partial y*\partial x*}\right)\right]\right\}\\
&+\frac{\alpha_2}{\rho_f}\frac{\partial}{\partial y^*}\left[4\left(\frac{\partial v^*}{\partial y^*}\right)^2+\left(\frac{\partial u^*}{\partial y^*}+\frac{\partial v^*}{\partial x^*}\right)^2\right],
\end{aligned}\tag{3}$$

$$u^*\frac{\partial T^*}{\partial x^*}+v^*\frac{\partial T^*}{\partial y^*}=\alpha^*\nabla^{*^2}T^*+\frac{(\rho c)_p}{(\rho c)_f}\left[D_B\nabla^*C^*.\nabla^*T^*+\frac{D_T}{T_\infty}\nabla^*T^*.\nabla^*T^*\right], \tag{4}$$

$$\left[u^*\frac{\partial C^*}{\partial x^*}+v^*\frac{\partial C^*}{\partial y^*}\right]=D_B\nabla^{*^2}C^*+\frac{D_T}{T_\infty}\nabla^{*^2}T^*, \tag{5}$$

with the following conditions

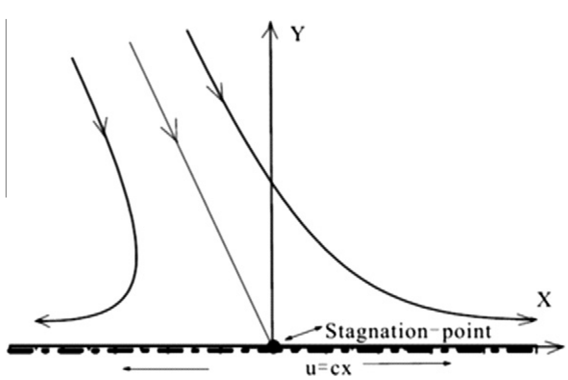

Fig. 1a. A sketch of the physical problem.

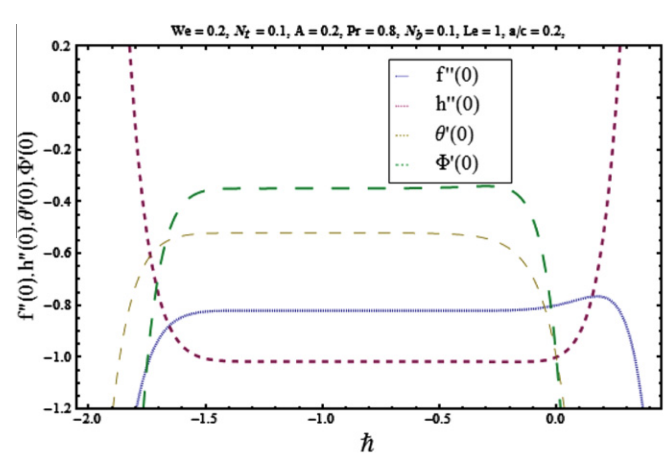

Fig. 1b. The ħ-curves of $f''(0)$, $h''(0)$, $\theta'(0)$ and $\Phi'(0)$ at 12th-order of approximations.