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Fully developed forced convective heat transfer in an annulus partially filled with metallic foams: An analytical solution

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ABSTRACT

An analytical solution for fully developed forced convective heat transfer in an annulus partially filled with metallic foam was proposed. The inner surface attached with an annular metallic foam layer was exposed to constant heat flux while the outer surface was adiabatic. In the metallic foam region, the Brinkman–Darcy equation was used to describe the fluid flow and the thermal non-equilibrium model was employed to establish the heat transfer equations. At the porous-fluid interface, no-slip coupling conditions were utilized to couple flow and heat transfer of the porous and open regions. A closed-form analytical solution was obtained for velocity and temperature profiles. The explicit form of friction factor and the Nusselt (*Nu*) number were also provided. The solutions were validated by two extreme cases: the empty annulus and the annulus fully filled with metallic foam. The effects of key parameters on friction factor, *Nu* number, and $j/f^{1/3}$ were examined. The relationship between flow heterogeneity and heat transfer was also discussed by introducing the flow heterogeneity coefficient. The porosity, pore density, and foam thickness for engineering applications were recommended. In the present analytical solution, a benchmark was also established for improving discretizing schemes in numerical works.

1. Introduction

The attractive thermal performance of open-cell metallic foam gives it great applied potential as catalyst support, burn rate enhancer for solid propellants, thermal storage, solar collector, fuel cell, electronics cooler, and for use in the aerospace and defense industries [1–3]. In particular, the light weight, high strength, high surface area density, high solid thermal conductivity, and good flow mixing capability of open-cell metallic foam make it effective for implementing various types of compact heat exchangers and heat sinks, which provides scientific problems for convective heat transfer.

Numerous works for flow and heat transfer in several common geometries fully filled with porous media have been published [2–6]. Using the Brinkman–Darcy and two-equation models, Lu et al. [2] presented an analytical solution for a metal–foam fully filled inner-tube while Zhao et al. [3] analytically investigated the metal–foam fully filled annulus for tube-in-tube heat exchangers. Du et al. [4] conducted a numerical study on parallel flow in metal–foam fully filled double-pipe heat exchangers using the two-equation model. Lee and Vafai [5] presented the analytical solution for forced convection heat transfer in parallel-plate channel fully filled with porous media and pointed out that the two-equation model is more accurate than the one-equation model for determining the large difference between solid and fluid thermal conductivities. Yang and Vafai [6] analytically studied a similar problem, but with both the uniform heat flux and uniform wall temperature boundary conditions. Overall, heat transfer in a duct fully fitted with porous media can be substantially enhanced. However, the corresponding pressure loss is too high, generally three to four orders of magnitude higher than that of the empty duct or more. This condition impedes the application of metallic foams with excellent thermal performance for most engineering cases requiring low pressure loss.

To enhance heat transfer with less pressure drop penalty, flow and heat transfer in a duct partially filled with porous media have been investigated to some extent [7–11]. Ming et al. [7] carried out a numerical study on heat transfer in a tube with a porous core and found that the partly porous duct may be applied for heat transfer enhancement. Poulikakos and Kazmierczak [8] analytically studied flow and heat transfer in two geometries, a parallel-plate channel and a circular tube, which were both partially filled with porous media. Gong et al. [9] performed similar work for the porous media partially filled tube by considering the thermal dispersion effect. Chikh et al. [10] did an analytical treatment of forced convective heat transfer in an annulus partially filled with porous media. Vafai

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Nomenclature

a_{sf}	specific surface area, m^{-1}
A	area, m^2
c	specific heat, $J \cdot kg^{-1} \cdot K^{-1}$
d_f	fiber diameter, m, $d_f = 1.18\sqrt{(1-\varepsilon)/(3\pi)}(1-\exp((\varepsilon-1)/0.04))$
d_p	pore diameter, m, $d_p = 0.0254/\omega$
Da	Darcy number
f	friction factor
h	heat transfer coefficient, $W \cdot m^{-2} \cdot K^{-1}$
h_{sf}	local convective heat transfer coefficient, $W \cdot m^{-2} \cdot K^{-1}$
K	permeability, m^2
k	thermal conductivity, $W \cdot m^{-1} \cdot K^{-1}$
k_r	thermal conductivity ratio, $k_r = k_f/k_s$
$\vec{n}_{p,f}$	unit vector normal to the porous-fluid interface pointing to the fluid side
Nu	Nusselt number
p	pressure, $N \cdot m^{-2}$
P	dimensionless pressure drop
Pr	Prandtl number
q	heat flux, $W \cdot m^{-2}$
q_m	mass flow, $kg \cdot s^{-1}$
r	radius, m
r_1	inner radius, m
r_2	outer radius, m
r_i	interfacial radius, m
R	dimensionless radius, $R = r/r_1$
R_2	dimensionless outer radius, $R_2 = r_2/r_1$
R_i	dimensionless interfacial radius, $R_i = r_i/r_1$
Re	Reynolds number, $Re = \rho_f u \cdot 2r_0/\mu_f$
Re_K	permeability Reynolds number, $Re_K = \rho_f u\sqrt{K}/\mu_f$
s	dimensionless factor in Eq. (19b)
t	dimensionless factor in Eq. (19b)
T	temperature, K
u	velocity, $m \cdot s^{-1}$
u_m	mean velocity, $m \cdot s^{-1}$
U	dimensionless velocity
x	axial position, m

Greek symbols

ε	porosity
η	dimensionless position of the annulus regulated from 0 to 1
θ	dimensionless temperature
μ	dynamic viscosity, $kg \cdot m^{-1} \cdot s^{-1}$
ξ	flow heterogeneity coefficient
ρ	density, $kg \cdot m^{-3}$
φ	polar angle, rad
ω	pore density, PPI (pores per inch)

Subscripts

b	bulk
e	effective
f	fluid
i	interface
m	mean
p	porous
r	ratio
s	solid
w	wall

and Thiyagaraja [11] obtained perturbation solution for three kinds of flow and heat transfer: porous-fluid interface, porous-solid interface and the interface of two porous media with different porosities. All of the works mentioned [7–11] were performed using one-equation model for duct partially filled with porous media.

According to Lee and Vafai [5], the two-equation model is more accurate than the one-equation model for metallic foams with high solid thermal conductivities, which calls for interfacial coupling conditions for non-equilibrium heat transfer in a foam–fluid system. The implementation of the two-equation model in a system with a porous-fluid interface increases the number of variables, temperature equations, and interfacial coupling conditions, making the analytical solution more complex for partially filled channels or ducts. To the authors' knowledge, only a few numerical [12–14] or theoretical [15] papers have been published for non-equilibrium heat transfer in a porous-fluid system. For non-equilibrium heat transfer in a porous-fluid system, Xu et al. [16,17] performed an investigation on the configurations of parallel-plate channels and single-pipes partially filled with metallic foams, and obtained closed-form analytical solution results. Yang and Vafai [18] presented a thermal condition for forced convective heat transfer in a tube with a porous core. For a system with both porous media and clear fluid, the interfacial coupling conditions can be categorized into two types: slip and no-slip conditions [19,20]. Alazmi and Vafai [21] found that the velocity and temperature profiles for slip and no-slip interfacial conditions are quite close, and the no-slip coupling condition is well accepted in the analytical process.

The annular duct is a common geometry for heat exchangers or heat sinks. No analytical work for non-equilibrium forced convective heat transfer using annular duct partially filled with metallic foams has been published in open literature. The current study aims to investigate analytically the fluid flow and heat transfer in metal–foam partially filled annulus. Normalized analytical solution on the effects of important parameters on flow and heat transfer are comprehensively analyzed and presented together with some useful proposals for practical applications.

2. Mathematical model

2.1. Physical description

These configuration details for metallic foam partially filled annulus are shown in Fig. 1. The annulus with inner radius (r_1) and outer radius (r_2) is partially filled with metallic foam. The metallic foam with outer-edge radius (r_i) is sintered on the inner wall and extends toward the outer wall. The inner wall of the duct is subjected to a constant heat flux (q_w) while its outer wall is adiabatic. Single-phase fluid with mean velocity (u_m) flows through the annular duct, removing heat imposed on the inner wall.

In this work, the fully developed laminar flow and heat transfer for incompressible fluid with constant thermal-physical properties are assumed while natural convection and thermal radiation are both disregarded. The metallic foam employed in the annular duct is both homogeneous and isotropic.

2.2. Governing equations

Based on the assumptions, the momentum and energy equations in the foam and fluid regions of the metal–foam partially