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# An advanced LMI-based-LQR design for voltage control of grid-connected wind farm

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**Abstract**

This paper presents an innovative voltage control scheme for a grid-connected wind farm. In wind power generation systems, operating conditions are changing continually by wind speed fluctuations and load changes. Therefore, a robust control mechanism is necessary. To enable a linear and robust control framework, the overall system is represented by a set of reduced-order linear systems that cover an operating range of interest determined by variations of the load. A control-design technique known as the linear quadratic regulator (LQR) can be conveniently utilized for multi-input multi-output systems. However, to make this approach applicable simultaneously to several linear systems, the LQR problem needs to be reformulated for finding a common Lyapunov function for the set of considered linear systems. This is accomplished by representing the underlying control optimization problem in terms of a system of linear-matrix-inequality (LMI) constraints and matrix equations that are simultaneously solved. The solution of LMI equations involves a form of quadratic Lyapunov function that not only gives the stability property of the controlled system but can also be used for achieving certain performance specifications. In addition, to make the control design applicable to realistic systems, with noise and disturbances in the measured signals, we consider a state observer. A candidate wind farm site on Vancouver Island, Canada, is conducted for simulation study. The proposed methodology is also flexible and readily applicable to larger wind farms of different configurations.


*Keywords:* Linear matrix inequality; Linear quadratic regulator; Observer; Voltage control; Wind farm

## 1. Introduction

As the present tendency of incorporating wind turbines (WTs) into large wind farms (WFs) continues, new possibilities for integrated design of individual turbines, the infrastructure within the WF, and the grid-connection interface open up [1]. Modern variable-speed WTs utilize power electronic converters for the grid connection and improved performance. By appropriately controlling the converters, it becomes possible to locally maintain the power factor (power factor control mode, PFC) or the voltage (local voltage control mode, LVC) [2,3]. As wind power penetration increases, the PFC and LVC modes are frequently not sufficient to achieve the desired voltage control, especially during events, such as faults [1,2], and may still require installation of additional devices (SVCs, switched-capacitors, transformer tap changers, etc.) to meet the power quality specifications [1,2,4–6]. However, there are always costs associated with the installation and operation of supplementary devices, which makes this option less attractable. Therefore, to achieve easier grid integration and reliable voltage control, alternative active voltage control of WTs is required. Furthermore, in modern WF applications, the WFs have to contribute voltage control within a specified allowable voltage level at a specified remote location—a point of common coupling (PCC).

Since in wind power generation systems, operating conditions are changing continually by wind speed fluctuations and load changes, a robust control mechanism is clearly necessary. This paper considers the linear quadratic regulator (LQR) approach as a framework for tuning the controller gains as this methodology is general and flexible, and can be formulated in terms of a performance-based optimization problem, for which the numerical solution techniques and software tools are widely available [7]. At the same time, the cost function (function to be minimized) may be defined in a number of ways that can simultaneously include several performance-based criteria. Another

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important advantage of using the LQR is that it can be formulated for the case when the overall plant is described by a set of linear systems that span a particular range of operating conditions.

However, to make this approach applicable simultaneously to several linear systems, the LQR problem needs to be reformulated for finding a common Lyapunov function for the set of considered linear systems. This is accomplished by representing the underlying control optimization problem in terms of a system of linear-matrix-inequality (LMI) constraints and matrix equations that are simultaneously solved. The solution of LMI equations involves a form of quadratic Lyapunov function that not only gives the stability property of the controlled system but can also be used for achieving certain performance specifications. As a powerful control-design tool, LMI techniques have been paid attention to power systems for the application of a robust tuning of controllers [8–17].

This paper proposes an advanced LMI representation of LQR that includes the cross-product terms in the cost function for the minimization of a quadratic performance measure. The motivation for adding the cross-product terms in the cost function is associated with the case when additional closed-loop regulation is required to maintain as closely as possible the optimal trajectory in the presence of disturbances and/or noise that cause small perturbations from the trajectory [18].

The proposed voltage control scheme with the advanced LMI-based-LQR (ALQR) is applied to a candidate WF site on Vancouver Island, Canada [19].

The paper is organized as follows: Section 2 covers the proposed innovative voltage control scheme; the advanced LMI representation of LQR is presented in Section 3; Section 4 describes the controller design; simulation is conducted in Section 5; the conclusion is drawn in Section 6.

## 2. Voltage control scheme of wind farm

Fig. 1 shows a simplified diagram of system considered in this paper [19]. Here, only three WTs are included to represent possible dynamic interaction among the individual turbines on the WF. Each WT is equipped with a step-up 0.69/34.5 kV transformer (TR). The WTs are connected in a chain using 9 km cables. The WF and the utility grid are connected through the 132 kV transmission line (TL, 100 km). Here, the utility grid is represented by a large synchronous generator (SG) [6]. The individual components of electrical subsystem including transformers, cables (C), and transmission lines are modeled using the *qd*-synchronous reference frame [20]. All electrical machines are also represented using full-order *qd*-models. In this paper, the GE 3.6 MW WT is considered [21]. The WT consists of the following components: a three-bladed rotor with the corresponding pitch controller; a mechanical gearbox; and a DFIG with the back-to-back voltage source converter (VSC) and the filter [21–23]. The model of entire system was implemented in detail using Simulink [24]. The operating condition of the given study system can be found in Appendix A.

As depicted in Fig. 2, the proposed voltage control scheme is to regulate the voltage at the PCC using multiple WTs. In a realistic WF, each WT may have somewhat different instantaneous wind speed and output of real power. Consequently, the availability of reactive power generation by each WT is also different. Thus, each WT should be included as a separate module in the overall model of the system.

When controlling multiple turbines, it is important that the operating limits of each individual WT are not exceeded. Assuming a proportional distribution, the portion of reactive power required from an individual VSC can be computed as

$$Q_{j,\mathrm{g}}^{\mathrm{set}} = \min\left\{ Q_{j,\mathrm{c}}^{\max},\ \frac{Q_{j,\mathrm{c}}^{\max}}{Q_{1,\mathrm{c}}^{\max} + Q_{2,\mathrm{c}}^{\max} + Q_{3,\mathrm{c}}^{\max}} \Delta Q_{\mathrm{pcc}} \right\} \tag{1}$$

where $j = 1, 2, 3$, $Q_{j,\mathrm{c}}^{\max}$ is the maximum reactive power (limit) that the $j$th VSC can provide, and $\Delta Q_{\mathrm{pcc}}$ is the total reactive power, which is required to support the voltage at the PCC. The maximum available reactive power from the combined back-to-back VSC can be expressed as

$$Q_{j,\mathrm{c}}^{\max} = 2\sqrt{(S_{j,\mathrm{c}}^{\max})^2 - P_{j,\mathrm{c}}^2} - Q_{j,\mathrm{r}} \tag{2}$$

where it is assumed that reactive power, which has been supplied to the DFIG is $Q_{j,\mathrm{r}}$, $P_{j,\mathrm{c}}$ is the real power of the converter and $-S_j^{\max} \le P_{j,\mathrm{c}} \le S_j^{\max}$ and the nominal apparent power of the converter is $S_{j,\mathrm{c}}^{\max}$ defined here as 1/3 of the WT rating [2].

## 3. LMI representation of LQR

This section presents an advanced LMI representation of LQR as a frame to design a controller as indicated in Fig. 2.

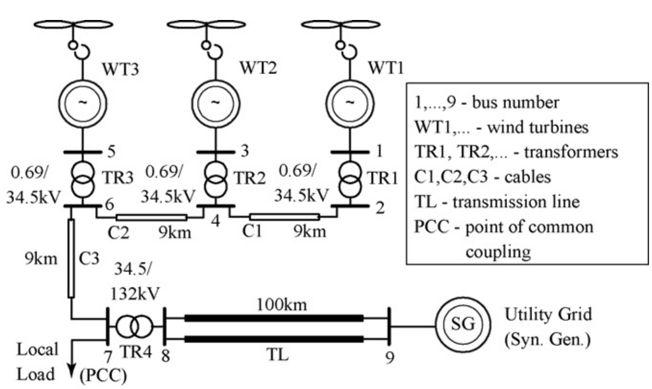


Fig. 1. Grid-connected wind farm system.

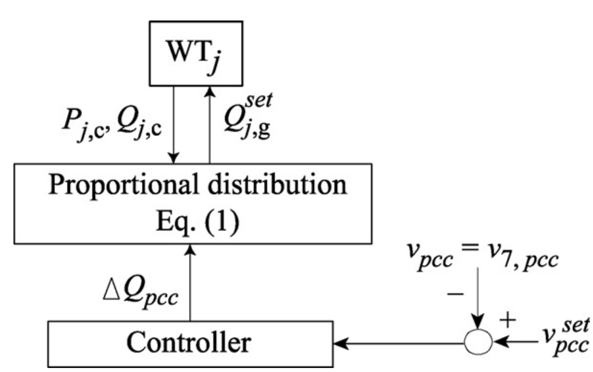


Fig. 2. Schematic diagram of the proposed voltage control.