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Dynamics and heat transfer in a quasi-two-dimensional MHD flow past a circular cylinder in a duct at high Hartmann number

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ABSTRACT

The fluid flow and heat transfer of a liquid metal past a circular cylinder in a rectangular duct (width-to-height aspect ratio of 2) under a strong transverse magnetic field is studied numerically using a quasi-two-dimensional model. Transition from steady to unsteady flow regimes is determined as a function of Hartmann number and blockage ratio, as are Strouhal number, and the heat transfer from the heated wall to the fluid. Downstream cross-stream mixing induced by the cylinder wake was found to increase heat transfer by more than a factor of two in some cases.

1. Introduction

The study of liquid metal flow in ducts in the presence of a transverse magnetic field has received attention because of its application to important technologies such as metallurgical processing, hydrodynamic generators, pumps, and blood flow meters. The primary application motivating this study is magnetic confinement fusion reactors, where liquid metal is used as a coolant and as a breeder material [1]. In most fusion reactor blankets the liquid metal circulates in an electrically insulated duct perpendicular to the applied magnetic field. The motion of liquid metal in a strong magnetic field induces electric currents, which in turn interact with the magnetic field resulting in a Lorentz force. This has a significant effect on the velocity distribution and the turbulence characteristics, and exerts a retarding force on the flow.

Under a strong magnetic field the flow is characterized by a laminar flow structure as velocity fluctuations in the direction of the magnetic field are strongly damped. Therefore, the heat transfer in the ducts of the first wall where large amount of heating power must be removed is dramatically decreased [2]. However, the two-dimensional turbulence that consists of vortices with axes parallel to the magnetic field are not significantly damped [3]. This turbulence could be used to enhance the heat transfer by using turbulence promoters such as a circular cylinder placed inside the duct of a blanket.

In the absence of a magnetic field, the flow past a circular cylinder depends on the Reynolds number and whether the flow is confined or unconfined. In an unconfined domain, the flow depends on Reynolds number as the sole control parameter. As Reynolds number increases, the flow becomes unstable first to two-dimensional disturbances, then three-dimensional instability modes [4]. At Reynolds numbers $Re \lesssim 46$, the flow is steady, and the wake comprises a counter-rotating pair of vortices attached to the rear of the cylinder. Beyond $Re \approx 46$, this flow becomes temporally unstable, with vortices shed periodically from alternating sides of the cylinder—the classical Kármán vortex street. This regime continues up to $Re \approx 180$, beyond which the flow becomes three-dimensional (for details see Williamson [4], Zdravkovich [5]).

In contrast, when the circular cylinder is confined in a plane channel, the nature and stability of the resulting flow changes significantly. The effect of wall confinement on the stability of the flow and flow parameters such as drag and lift coefficients, Strouhal number, and Nusselt number have been investigated by Coutanceau and Bouard [6], Chen et al. [7], Behr et al. [8], Anagnostopoulos et al. [9], Zovatto and Pedrizzetti [10], Khan et al. [11], Chakraborty et al. [12], Sahin and Owens [13], Cliffe and Tavener [14], Mettu et al. [15]. In this configuration, the blockage ratio (cylinder diameter to channel width) is an additional control parameter. Due to the additional dissipation at the confining walls, the transition from steady to unsteady flow is delayed and occurs at Reynolds numbers higher than that in unconfined flow.

For the liquid metals used in the fusion blanket, the Hartmann number (*Ha*, the ratio between electromagnetic and viscous forces) and the interaction parameter ($N = Ha^2/Re$, the ratio of electromagnetic to inertial forces) are very high (i.e. $Ha \gg 1, N \gg 1$). This implies that the electromagnetic forces outweigh the viscous forces, and the flow in a rectangular duct can be split into one or

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Nomenclature

a	channel half-height
A	instability mode amplitude
B	magnetic field
C_d	cylinder drag coefficient
C_l	cylinder lift coefficient
d	cylinder diameter
f	wake oscillation frequency
F'_d	drag force per unit span
h	channel width
Ha	Hartmann number
HI	percentage heat transfer increment
L	length along heated bottom wall
N	interaction parameter
N_{el}	number of spectral elements in mesh
Nu	time-averaged Nusselt number
Nu_0	time-averaged Nusselt number in empty channel
Nu_w	local Nusselt number
$\overline{Nu}_w$	time-average of local Nusselt number
ΔP	pressure drop across channel
p	pressure
Pe	Peclet number
Re	Reynolds number
Re_c	critical Reynolds number for onset of vortex shedding
St	Strouhal number
St_c	Strouhal number at Re_c
ΔT	temperature difference between channel side walls
t	time
T	temperature field
T_f	bulk fluid temperature
T_o	temperature of fluid entering channel
T_w	temperature of hot channel side-wall
u	streamwise component of velocity
$\mathbf{u}$	velocity vector field
U_o	peak inlet velocity
x	Cartesian coordinate in flow direction
x_u	length to upstream boundary
x_d	length to downstream boundary
y	Cartesian coordinate across channel

Greek symbol

β	blockage ratio
δ_H	Hartmann boundary layer thickness
δ_S	Shircliff boundary layer thickness
ν	kinematic viscosity
ρ	fluid density
σ	magnetic permeability
ω	vorticity in x–y plane

more quasi-inviscid core flows, separated by thin boundary or shear layers of two types. The core occupies almost the whole duct cross section. In this region there is a balance between the pressure and the Lorentz force. In the boundary layer region, viscous forces are balanced by Lorentz forces. The boundary layer adjacent to the wall perpendicular to the magnetic field is known as the Hartmann layer, and is characterized by an exponential decay of the velocity towards the wall. Its thickness scales as $\delta_H \sim O(Ha^{-1})$ [16]. The boundary layer adjacent to the wall parallel to the magnetic field is known as a Shercliff layer (or side layer), with a thickness that scales with $\delta_S \sim Ha^{-1/2}$ [16].

The magnetic field may intensify vortices parallel to the magnetic field. The effect of the magnetic field on the evolution of the vortices has been investigated theoretically by Davidson [17], Pothherat et al. [18], Sommeria and Moreau [19] and experimentally by Alboussiere et al. [20], Andreev and Kolesnikov [21], Burr et al. [22], Klein et al. [23], Buhler [24]. The properties of such flows was first studied by Kolesnikov and Tsinober [25] in decaying grid turbulence, and later by Moreau and Sommeria [26] in the electromagnetically forced regimes. The mechanisms are explained by Sommeria and Moreau [19], who showed that the velocity components perpendicular to the magnetic field are dissipated by strong joule heating dissipation, except in the Hartmann layer where the turbulent fluctuations are weakly damped in that direction.

In order to initiate turbulence, Buhler [24] numerically investigated the transition between stationary flow and quasi two-dimensional vortex structures for the shear flow profile which is generated by the insertion two thin strips of different conductivity in the transverse and streamwise directions. The resulting flow profile was similar to the Kármán vortex street behind a circular cylinder without a magnetic field. However, another mechanism for producing vortices in an insulated channel is by using turbulence promoters such as cylindrical or rectangular obstacles placed parallel to the direction of the applied magnetic field. This approach has been investigated experimentally by Kit et al. [27] and Kolesnikov and Tsinober [28]. The shape of vortex street in an open channel with only one Hartmann wall under a strong magnetic field has been investigated experimentally by Papailiou [29]. He observed that under the effect of a magnetic field, the wake becomes narrower than that without a magnetic field. In addition, it has been shown that the distance between the vortices in the main stream direction was reduced and a row of vortices were organized downstream. However, the effect of the magnetic field aligned with the cylinder axis on Strouhal number was weak.

More recently, Frank et al. [30] investigated experimentally the wake behind a circular cylinder of quasi two-dimensional liquid metal flow in a rectangular duct for the condition of high Reynolds and Hartmann number. The value of Reynolds number was within the range $100 \leqslant Re \leqslant 10{,}000$, and for the Hartmann number $500 \leqslant Ha \leqslant 1200$, which gave values of interaction parameter N between 10 and 576. The blockage ratio was set to $\beta = 0.1$. The experimental results revealed that there is a linear dependence of the critical Reynolds number on Hartmann number for the onset of time-dependent vortex shedding. They conclude that the dissipation of *vortical energy* is achieved by the quasi two-dimensional linear damping term, which leads to a continuous decrease in vortex intensity with downstream distance.

For the same configuration Muck et al. [31] have performed three-dimensional numerical simulations for liquid metal flows around a square cylinder in a rectangular duct. The interaction parameter was varied in the range $0 \leqslant N \leqslant 36$ and the Hartmann number in the range $0 \leqslant Ha \leqslant 85$. The numerical simulation was carried out for two fixed values of Reynolds number, 200 and 250. The numerical results were in a good agreement with the experimental obtained by Frank et al. [30].

Dousset and Potherat [32] studied numerically the wake behind a circular cylinder for a quasi two-dimensional liquid metal flow placed in a square duct under a strong magnetic field parallel to the cylinder axis. Their simulations were carried out for Reynolds numbers $Re \leqslant 6000$, Hartmann numbers $0 \leqslant Ha \leqslant 2160$, and blockage ratio $\beta = 0.25$. The results revealed that the transition between the flow regimes were controlled by the linear friction term proportional to Re/Ha. Also, it has been found that the break up of the two-dimensional vortex street occurs at $Re = 5000$.