International Journal of Heat and Mass Transfer 54 (2011) 1506–1519

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Inverse analysis with integral transformed temperature fields: Identification of thermophysical properties in heterogeneous media

Carolina P. Naveira-Cotta, Renato M. Cotta *, Helcio R.B. Orlande

Laboratory of Transmission and Technology of Heat-LTTC, Mechanical Eng. Dept. – POLI & COPPE/UFRJ, Universidade Federal do Rio de Janeiro, Rio de Janeiro, RJ, CEP.21945-970, Brazil

ARTICLE INFO

Article history:
Received 28 April 2010
Received in revised form 11 November 2010
Accepted 11 November 2010
Available online 7 January 2011

Keywords:
Integral transforms
Heterogeneous media
Heat conduction
Inverse problem
Thermophysical properties
Bayesian inference

ABSTRACT

The objective of this work is to introduce the use of integral transformed temperature measured data for the solution of inverse heat transfer problems, instead of the common local transient temperature measurements. The proposed approach is capable of significantly compressing the measured data through the integral transformation, without losing the information contained in the measurements and required for the solution of the inverse problem. The data compression is of special interest for modern measurement techniques, such as the infrared thermography, that allows for fine spatial resolutions and large frequencies, possibly resulting on a very large amount of measured data. In order to critically address the use of integral transformed measurements, we examine in this paper the simultaneous estimation of spatially variable thermal conductivity and thermal diffusivity in one-dimensional heat conduction within heterogeneous media. The direct problem solution is analytically obtained via integral transforms and the related eigenvalue problem is solved by the Generalized Integral Transform Technique (GITT). The inverse problem is handled with Bayesian inference by employing a Markov Chain Monte Carlo (MCMC) method. The unknown functions appearing in the formulation are expanded in terms of eigenfunctions as well, so that the unknown parameters become the corresponding series coefficients. Such projection of the functions in an infinite dimensional space onto a parametric space of finite dimension also permits that several quantities appearing in the solution of the direct problem be analytically computed. Simulated measurements are used in the inverse analysis; they are assumed to be additive, uncorrelated, normally distributed, with zero means and known covariances. Both Gaussian and non-informative uniform distributions are used as priors for demonstrating the robustness of the estimation procedure.

1. Introduction

The analysis of diffusion problems in heterogeneous media involves formulations with spatial variations of the thermophysical properties in different ways, such as large scale variations in functionally graded materials (FGM), abrupt variations in layered composites, and random variations due to local concentration fluctuations in dispersed phase systems [1–6]. For instance, composite materials have been providing engineers with increased opportunities for tailoring structures to meet a variety of property and performance requirements. As the composite material morphology in applications presents endless possibilities due to design and manufacturing processes, the characterization of their physical properties is to be made almost case to case [7–13].

The accurate representation of the heat conduction phenomena requires a detailed local solution of the temperature distribution, generally with the aid of discrete numerical solutions with sufficient mesh refinement and computational effort and/or semi-analytical approaches for specific or simplified functional forms. Analytical solutions of linear diffusion problems have been analyzed and compiled in [14], where seven different classes of heat and mass diffusion formulations are systematically solved by the classical Integral Transform Method. The obtained formal solutions are applicable over a very broad range of problems in heat and mass transfer, in part illustrated in the referred compendium. Later on, the classical integral transform approach gained a hybrid numerical–analytical implementation and is in general referred to as the Generalized Integral Transform Technique (GITT) [15–21], offering more flexibility in handling non-transformable problems, including, among others, the analysis of nonlinear diffusion and convection–diffusion problems.

The usefulness of such direct problem solutions is nevertheless limited by the precise knowledge of the corresponding thermophysical properties and boundary condition coefficients that are fed in the corresponding models, and quite often need to be determined by the appropriate inverse problem analysis [22–29]. Among the various available solution techniques of inverse problems [30–34], a fairly common approach is related to the

* Corresponding author.
E-mail addresses: cpncotta@hotmail.com (C.P. Naveira-Cotta), cotta@mecanica.coppe.ufrj.br (R.M. Cotta), helcio@mecanica.coppe.ufrj.br (H.R.B. Orlande).

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doi:10.1016/j.ijheatmasstransfer.2010.11.042

Nomenclature

a	coefficient in time lag function of applied heat flux, Eq. (24c)
b	coefficient in time lag function of applied heat flux, Eq. (24c)
c_p	specific heat, Eq. (1.a)
$d(x)$	linear dissipation operator coefficient, Eq. (3.a)
$f(t)$	time lag function in applied heat flux, Eq. (24a)
$h_{\text{eff}}(x)$	effective heat transfer coefficient, Eq. (1.a)
$k(x)$	space variable thermal conductivity, Eq. (1.a)
L_x	plate length
L_z	plate thickness
M	truncation order in eigenvalue problem expansion
M_n	normalization integrals in auxiliary eigenvalue problem
N_T	truncation order in temperature expansion
N_w, N_k	truncation orders in coefficients expansions, $w(x)$ and $k(x)$, respectively
N_{Fk}, N_{Fw}, N_{Fd}	number of parameters to be estimated in each filtered solution, $wf(x)$, $kf(x)$ and $df(x)$, respectively
N_{Pk}, N_{Pw}, N_{Pd}	number of parameters to be estimated in each parametrization, $w(x)$, $k(x)$ and $d(x)$, respectively
N_f	number of parameters to be estimated in time behavior of the applied heat flux, $f(t)$
N_P	number of parameters to be estimated, Eq. (20b)
N_x	number of measurements along the spatial domain (sensors)
N_t	number of measurements in time
N_m	total number of measurements
N_i	normalization integrals in original eigenvalue problem
$P(x,t)$	source term, Eq. (3.a,g)
$q_w(x,t)$	applied heat flux, Eq. (1.a)
q_{inf}	heat flux dissipated from electrical resistance, Eq. (24.b)
t	time variable
$T_m(x,t)$	temperature distribution
$w(x)$	thermal capacity, Eq. (3.a)
$w_f(x)$	filter for thermal capacity expansion
x	space coordinate
$\mathbf{Y}$	vector of measurements
$\mathbf{P}$	vector of unknown parameters
$\mathbf{P_w}$, $\mathbf{P_k}$, $\mathbf{P_d}$, $\mathbf{P_f}$	vector of unknown parameters for $w(x)$, $k(x)$, $d(x)$ and $f(t)$ respectively
$\mathbf{T}$	vector of estimated temperatures
$\mathbf{W}$	covariance matrix of the measurement errors

Greek symbols

γ	parameter in heat flux or linear dissipation coefficient spatial variation
ε	emissivity
λ	eigenvalues of the auxiliary problem
μ	eigenvalues of the original problem
ψ	eigenfunctions of the original problem
Ω	eigenfunctions of the auxiliary problem
ρ	density

Subscripts and Superscripts

i, n, m	order of eigenquantities
$-$	integral transform
$\sim$	normalized eigenfunction
d	dispersed phase (filler) properties
f	filtering function in the coefficient expansion
m	matrix phase properties

minimization of an objective function that usually involves the quadratic difference between measured and estimated dependent variables, such as the least squares norm, or its modified versions with the addition of regularization terms. Although very popular and useful in many situations, the minimization of the least squares norm is a non-Bayesian estimator. A Bayesian estimator is basically concerned with the analysis of the posterior probability density, which is the conditional probability of the parameters given the measurements, while the likelihood is the conditional probability of the measurements given the parameters [33]. If we assume the parameters and the measurement errors to be independent Gaussian random variables, with known means and covariance matrices, and that the measurement errors are additive, a closed form expression can be derived for the posterior probability density. In this case, the estimator that maximizes the posterior probability density can be recast in the form of a minimization problem involving the maximum a posteriori objective function. On the other hand, if different prior probability densities are assumed for the parameters, so that the Posterior Probability Distribution may not allow an analytical treatment, Markov Chain Monte Carlo (MCMC) methods are required to draw samples of all possible parameters, and thus inference on the posterior probability becomes inference on the samples.

In this work we use Bayesian inference for the estimation of spatially variable equation and boundary condition coefficients in diffusion problems, by employing the method of Markov Chain Monte Carlo (MCMC) [33,35–38]. The Metropolis-Hastings algorithm is employed for the sampling procedure [39,40], implemented in the *Mathematica* platform [41]. This sampling procedure used to recover the posterior distribution is in general the most expensive computational task in solving an inverse problem by Bayesian inference, since the direct problem is calculated for each state of the Markov Chain. In this context, the use of a fast, accurate and robust computational implementation of the direct solution [42] is extremely important. Thus, the integral transformation approach discussed above becomes very attractive for the combined use with the Bayesian estimation procedure, since all required expressions in the method are analytically obtained at once by symbolic computation and the single numerical repetitive task is the solution of an algebraic matrix eigenvalue problem [42–44]. Also, instead of seeking the function estimation in the form of a sequence of local values for the variable coefficients, an alternative approach is utilized based on the eigenfunction expansion of the functions to be estimated [42]. As a result, the solution of the inverse problem is performed in a finite dimensional space of parameters, involving the corresponding series coefficients.

The main contribution of the present work is the analysis of the inverse problem in the transformed temperature field, instead of employing the directly measured temperature data. The experimental temperature values at each time are integral transformed along the spatial domain to yield transformed temperature measurements of increasing order, which is the eigenvalue order of the auxiliary problem used in the transformation. This procedure is particularly advantageous when a substantial amount of experimental measurements are available, such as in thermographic sensors, thus permitting a remarkable data compression through the integral transformation process, without discarding any of the available measurements.

In order to demonstrate the applicability of the proposed estimation approach, a simulated experiment is used, which employs