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Developing flow in inclined laminar-laminar stratified systems: Investigation of the multiple holdup problem

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ABSTRACT

In this study, the multiple holdup solutions problem for stratified laminar-laminar flow in a channel is investigated. A stationary but developing monodimensional flow model is adopted here to follow the evolution of the holdup value from a non-equilibrium inlet condition to the final downstream and fully developed solution. A first order Ordinary Differential Equation (ODE) solver is used to perform this analysis under the assumption that the flow remains supercritical all along the pipe. The possibility of having a hydraulic shock during the longitudinal evolution of the system is investigated too. A second order ODE model is then proposed to handle situations with shocks, by including the effects of the longitudinal viscous stress diffusion when large interface level gradients occur. The results are also discussed regarding the approach of minimization of a potential function, showing a good consistency between the two methods.

1. Introduction

Gas-liquid or liquid-liquid stratified flow patterns are frequently encountered in many industrial applications, where because of gravity, the heavier fluid is flowing at the bottom part of the pipe while the lighter one above it. Usually, such a stratified flow is predicted using a 1D approach, presented by Taitel and Dukler (1976b). In this approach, the wall shear stress of each fluid and the interfacial shear stress, which are unknown *a priori*, must be estimated using closure laws. However, it is now recognized (Baker et al., 1988) that in certain operating conditions, this procedure can lead to a non-unique holdup (or heavier phase fraction) in the solutions. This typically occurs for upward inclined pipes and low flow rates of the heavier phase. Landman (1991) dedicated theoretical investigations on this issue and showed that the occurrence of multiple values for holdup (and consequently pressure drop) persists when considering the exact solutions of the Navier–Stokes equations for laminar-laminar flow in a channel. Ullmann et al. (2003) observed experimentally that at least 2 of the 3 solutions predicted by the model are feasible in such configuration and postulated that an hysteresis phenomenom could be involved. Thus leading to the question that which of these solutions will actually occur. This information is very relevant in practice for gas-condensate pipelines, where the liquid content flowing in the pipeline dictates the choice of the pipe diameter and the minimum gas flow-rate at which it can be operated.

Barnea and Taitel (1992) aimed at dealing with this problem using structural stability analysis of the different solutions. In that way, they used a transient set of equations of motion assuming a pipe uniformly filled all along its length with shear stress closure laws derived from the single-phase case. They concluded that, in upward inclined systems, the thinnest and the thickest steady-state solutions are linearly stable while the intermediate ones are unstable and would not be realized. From a non-linear stability analysis, it appears that only the lowest holdup solution is stable in response to finite disturbances. Landman (1991) pointed out that the assumption of spatial uniformity should be avoided to perform a linear stability analysis because of its inconsistency with mass conservation. On the other hand, the analysis presented in this paper assumes a stationary but developing laminar-laminar flow in an inclined channel. A simple model based on the first order derivative equation was used to follow the evolution of the holdup longitudinally through the pipe from different values fixed at the inlet.

In the present paper, the equations involved in the multiple holdup problem are reminded and discussed regarding the approach by the minimization of a potential presented in Thibault et al. (2015). Then, the results from the structural stability analysis are presented for different cases and analyzed by paying attention to the supercritical flow condition and the possibility of encountering a hydraulic shock.

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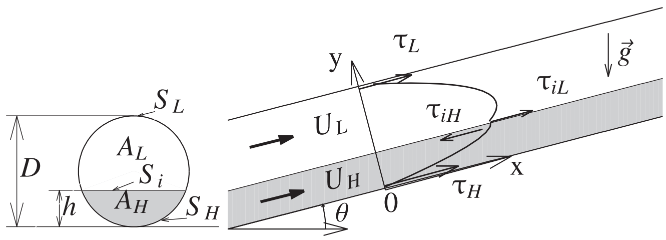

Fig. 1. Scheme of the stratified flow configuration.

2. Governing equations and modeling

The two-fluid one-dimensional integral equations used in the stratified flow model describing two immiscible fluids flowing in an inclined pipe are:

$$\frac{\partial}{\partial t}(\rho_k A_k) + \frac{\partial}{\partial x}(\rho_k A_k U_k) = 0 \tag{1}$$

for the mass conservation and

$$\frac{\partial}{\partial t}(\rho_k A_k U_k) + \frac{\partial}{\partial x}(\rho_k A_k \gamma_k U_k^2) = -\frac{\partial A_k P_k}{\partial x} + P_{i,k}\frac{\partial A_k}{\partial x} - \rho_k A_k g \sin(\theta) - \tau_k S_k - \tau_{ik} S_i + \frac{\partial}{\partial x}\left(\mu_k A_k \frac{\partial U_k}{\partial x}\right) \tag{2}$$

for the momentum conservation. Here A_k is the cross-sectional area of the phase k (denoted L for the light fluid and H for the heavy one), U_k its mean velocity (j_k being the superficial velocity), γ_k the velocity shape factors, S_k the wall perimeter, S_i the interfacial perimeter, ρ_k the density, τ_k and τ_{ik} the shear stresses exerted on the fluid k by the wall and the interface respectively (see Fig. 1). The longitudinal viscous diffusion term $\frac{\partial}{\partial x}(\mu_k A_k \frac{\partial U_k}{\partial x})$ is generally neglected in the literature which will be also the case here first. Nevertheless, a model introducing the influence of this term will be presented in the last part of the text.

P_k is the space averaged pressure in the section A_k and $P_{i,\,k}$ the pressure at the interface, which could be different in each phase due to surface tension effects. An hydrostatic evolution of the pressure in the section A_k is assumed here and the effect of surface tension is neglected ($P_{i,H} = P_{i,L}$). The interfacial shear stresses are defined by $\tau_i = -\tau_{iH} = \tau_{iL}$, and one can finally obtain the combined momentum equation:

$$\rho_H \frac{\partial U_H}{\partial t} - \rho_L \frac{\partial U_L}{\partial t} + \left[\rho_H(1-\gamma_H)\frac{U_H}{A_H} + \rho_L(1-\gamma_L)\frac{U_L}{A_L}\right]\frac{\partial A_H}{\partial t} + \rho_H U_H \frac{\partial \gamma_H U_H}{\partial x} - \rho_L U_L \frac{\partial \gamma_L U_L}{\partial x} + (\rho_H - \rho_L) g \cos(\theta)\frac{\partial h}{\partial x} = F \tag{3}$$

$$\text{with} \quad F = \tau_L \frac{S_L}{A_L} - \tau_H \frac{S_H}{A_H} + \tau_i S_i \left(\frac{1}{A_H} + \frac{1}{A_L}\right) - (\rho_H - \rho_L) g \sin(\theta) \tag{4}$$

and h the level of the heavy fluid. It is to be noted that the expression of the shear stresses should be evaluated in the general case and not only at fully-developed steady-state solutions ($F = 0$) to remain fully valid. However, the expressions for the shear stresses are derived in this study in a conventional way assuming a quasi-steady relation for the local holdup and phase velocities which is reasonable here since small gradients of the holdup will be considered in the following lines.

In the case of steady-state and fully developed laminar-laminar flow between two infinite parallel plates separated by a distance D, where the exact analytical expression of the velocity profiles can be easily obtained (Biberg and Halvorsen, 2000). Thus, there is no need to use approximative closure relations since the continuity of the velocity and the shear stress at the interface lead to the following expressions for the wall shear stresses:

$$\tau_H = \mu_H \frac{\partial u_H}{\partial y}\Big|_{y=0} = \underbrace{\frac{3\mu_H U_H}{\varepsilon D}}_{\textit{free surface flow}} - \underbrace{\frac{3\mu_H \mu_L}{\mu_H/\varepsilon + \mu_L/(1-\varepsilon)}\frac{U_L - U_H}{(1-\varepsilon)\varepsilon D}}_{\textit{shear flow}} \tag{5}$$

$$\tau_L = -\mu_L \frac{\partial u_L}{\partial y}\Big|_{y=D} = \underbrace{\frac{3\mu_L U_L}{(1-\varepsilon)D}}_{\textit{free surface flow}} + \underbrace{\frac{3\mu_H \mu_L}{\mu_H/\varepsilon + \mu_L/(1-\varepsilon)}\frac{U_L - U_H}{(1-\varepsilon)\varepsilon D}}_{\textit{shear flow}} \tag{6}$$

and for the interfacial shear stress:

$$\tau_i = 6\frac{\mu_H \mu_L}{\mu_H/\varepsilon + \mu_L/(1-\varepsilon)}\frac{U_L - U_H}{(1-\varepsilon)\varepsilon D} \tag{7}$$

where ε is the heavy fluid phase fraction so-called holdup (*i.e.* h/D) and $u_k(y)$ the local velocity distribution in phase k. It could be observed that the expression of the wall shear stresses make it possible to have a negative value for τ_H, while the mean velocity U_H remains positive. This would happen in the case of an interfacial shear stress two times larger than the free surface contribution. When this condition is met, a partial backflow would appear in the heavy phase near the bottom wall. Moreover, the velocity shape factors γ_k, which are easily obtained from the velocity profile (see Appendix A), are largely affected by partial backflow.

2.1. Multiple holdup problem

The steady-state solutions of the system ($F = 0$) depend on the viscosity ratio $\mu^* = \mu_H/\mu_L$, the Lockhart & Martinelli parameter X^2 (Lockhart and Martinelli, 1949) and the Taitel & Dukler inclination parameter Y (Taitel and Dukler, 1976a,b). These parameters are defined in a general case as follows:

$$X^2 = \frac{(\partial P/\partial x)_{H,S}}{(\partial P/\partial x)_{L,S}} \quad \text{and} \quad Y = \frac{\Delta\rho g \sin(\theta)}{(\partial P/\partial x)_{L,S}} \tag{8}$$

where, $(\partial P/\partial x)_{k,\,S}$ is the pressure drop observed if the phase k were flowing alone in the conduit, and $\Delta\rho = \rho_H - \rho_L$. Here, the Lockhart–Martinelli and inclination parameters are expressed as:

$$X^2 = \frac{\mu_H j_H}{\mu_L j_L} \quad \text{and} \quad Y = -\frac{\Delta\rho g D^2 \sin(\theta)}{12\mu_L j_L} \tag{9}$$

which lead to the holdup relation in its dimensional form:

$$F = (-\partial P/\partial x)_{L,S}\Big(Y - \big[X^2(1-\varepsilon)^2\big(\varepsilon(4-\varepsilon) + \mu^*(1-\varepsilon)^2\big) - \varepsilon^2\big(\mu^*(3-2\varepsilon) + \varepsilon^2(1-\mu^*)\big)\big]\Big/ \big[4\varepsilon^3(1-\varepsilon)^3(\varepsilon(1-\mu^*) + \mu^*)\big]\Big) = 0 \tag{10}$$

where the quantity $(-\partial P/\partial x)_{L,S} = 12\mu_L j_L/D^2$ is always positive by definition. Eq. (10) can exhibits one, two or three physically acceptable solutions (*i.e.* $0 < \varepsilon < 1$) depending on the control parameters X^2 and Y. The boundaries of the multiple solutions regions are defined by the following equations on Y and X^2:

$$X^2 = -\,\varepsilon^2\big[(1-\mu^*)^2\varepsilon^4 - 2\mu^*(1-\mu^*)\varepsilon^3 + 2\mu^*(3-4\mu^*)\varepsilon^2 - 2\mu^*(1-3\mu^*)\varepsilon - \mu^{*2}\big]\Big/\Big((1-\varepsilon)^2(\dagger)\Big) \tag{11}$$

and

$$Y = \frac{\Big((1-\mu^*)\varepsilon^2 + \mu^*\Big)\Big((1-\mu^*)\varepsilon^2 - 2(1-\mu^*)\varepsilon - \mu^*\Big)}{2\varepsilon(1-\varepsilon)^3(\dagger)} \tag{12}$$