International Journal of Multiphase Flow 85 (2016) 284–297

Contents lists available at ScienceDirect

International Journal of Multiphase Flow

journal homepage: www.elsevier.com/locate/ijmultiphaseflow

Development and validation of a reduced order history force model

Husam A. Elghannay, Danesh K. Tafti*

Department of Mechanical Engineering, Virginia Tech, Blacksburg, VA 24061, USA

ARTICLE INFO

Article history:
Received 6 October 2015
Revised 17 June 2016
Accepted 26 June 2016
Available online 29 June 2016

Keywords:
Particle dynamics
History/Basset force modeling
Point force models

ABSTRACT

The history force model accounts for temporal development in fluid gradients in the viscous region surrounding a particle in point particle methods. The calculation of the history force typically requires storing and using relative velocity information during the life time of the particle. For a large number of particles integrated over large times, history force calculation can become prohibitively expensive. The current work presents a new modeling approach to calculate the history force in which a decay function is applied to a stored cumulative value of the history force. The proposed formulation is equivalent to applying the same function obtained from a constant acceleration assumption to a running average of the acceleration within the memory time of the particle. The new force model is validated with experimental measurements of settling spheres at Reynolds numbers ranging from around one to a few hundreds and at density ratios from 1.2 to about 9.32. More validation work was carried-out with experimental measurements of oscillating spheres at different frequencies and amplitudes, as well as bouncing spheres at different Reynolds numbers and density ratios. The model shows very good agreement with the experiments of settling spheres and reasonable/good agreement with oscillating and bouncing sphere experiments. The proposed model significantly reduces the computational resources required to calculate the history force especially when large number of particles need to be integrated over long times.

1. Introduction

Point force models are widely used in simulating natural and industrial particulate flow systems because they allow for simulating relatively large number of particles for long times. In this approach, the fluid–particle interaction force is accounted by a linear combination of different force models. Among the different force models, history force is considered as the most computationally expensive as it typically requires storing and using relative velocity information during the life-time of the particle. The calculation of the history force using this approach, however, requires significant computing resources. For example, it would require 1.2 GB of memory to store a single precision value of relative velocity in three dimensions to simulate 100,000 particles for 1000 time steps (Dorgan and Loth, 2007). Moreover, the inclusion of the history force may render the entire computation impractical when large number of particles are simulated (González et al., 2006).

In part, the work on development of history force models was motivated by the need to efficiently model sediment transport (Niño and García 1994, 1998; González et al. 2006; Bombardelli et al., 2008). In one mode of sediment transport, sediment particles move by jumping along the channel bottom wall (bed of particles) or by taking projectile trajectories (saltating). Thus, researchers have extensively focused on simulating the motion of saltating particles to develop sediment transport formulas or to improve existing models of sediment transport (Lukerchenko, 2010; Bialik, 2011; Bialik et al., 2012; Bialik and Czernuszenko, 2013).

Though some researchers completely neglect the history force in their numerical modeling of solid particles in sediment transport simulations (e.g., Schmeeckle and Nelson 2003; Kholpanov and Ibyatov 2005; Lee et al., 2006; Schmeeckle, 2014), numerical and empirical evidence suggests that the history force plays an important role in the transport of relatively small sediment particles moving near the bed. Comparing laboratory observations with numerical results provides evidence that while the history force is negligible for *gravels* moving as bedload; it becomes extremely important for *sand* (Niño and García, 1994, 1998). The history force was found to be significant to correctly describe the particle trajectory for a high Reynolds number of 4000 (Mordant and Pinton 2000; Bombardelli et al., 2008), and appreciable for Reynolds numbers smaller than or of the order of one for large density ratios (Armenio and Fiorotto, 2001).The length and height of a single particle jump can be under-predicted by about 40 and 15%, re-

* Corresponding author at: Department of Mechanical Engineering, Virginia Tech, Goodwin Hall, (Room 213E) 635 Prices Fork Road (0238), Blacksburg, VA 24061, USA.

E-mail address: dtafti@exchange.vt.edu (D.K. Tafti).

http://dx.doi.org/10.1016/j.ijmultiphaseflow.2016.06.019
0301-9322/

spectively when the history force is neglected in the case of sand (Niño and García, 1998) and the cumulative effect of such differences for multiple jumps can lead to very large errors in predictions of sediment transport (Bombardelli et al., 2008).

Many researchers have acknowledged the fact that reducing the computational cost associated with the history force would be beneficial and have worked towards developing less computationally intensive methods for modeling the history/Basset force. Michaelides (1992) used Laplace transform to recast the linear equations of particle motion in a simplified creeping flow velocity field. He reported 6–11 times faster simulations with no need to store the velocity with the new method. His procedure, however, does not apply to non-linear equations or in random velocity fields (González et al., 2006; Bombardelli et al., 2008).

Gonzalez et al. (Gonzalez et al., 2006; Bombardelli et al., 2008) were the first to use a series expansion of the semi-derivative formula for the history force in creeping flow as proposed by Tatom (1988). They also introduced a "memory time period" during which the history of the particle affected the current particle motion. The use of the semi-derivative formulation was found to reduce the computational cost by 20% compared to conventional techniques, and a time reduction of 10–30% of the original simulation time was realized when the concept of memory time period was employed (Bombardelli et al., 2008).

The justification for the use of memory time lies in the fact that particle behavior at earlier times have a smaller contribution to the history force integral (Lukerchenko, 2010) at the current time. Mordant and Pinton (2000) noted that the history force at finite particle Reynolds number (as defined in Eq. (5)) can be well presented by the creeping flow expression for short intervals and becomes negligible thereafter (Dorgan and Loth, 2007). Such findings inspired the development of a window model in which the history force was integrated over a prescribed finite time using the Basset kernel (Dorgan and Loth, 2007). The truncation of integration time can yield more than an order of magnitude savings in CPU time compared to conventional history force calculations (Dorgan and Loth, 2007). The model performed well for settling spheres with Reynolds numbers ranging from 9 to 853 and density ratios from 1.17 to 9.32. For oscillating particles the window model produced reasonable results only when changes in relative particle acceleration over the integration window were limited (Dorgan and Loth, 2007). To further increase the accuracy of the window model, Van Hinsberg et al. (2011) proposed the use of an exponential function to represent the tail rather than truncating it.

Although the use of the window model significantly reduces the computational time and memory requirements for calculating the history force, depending on Reynolds number and time step used, the model still requires storing particle information for few hundreds of time steps. Such requirement is still considerable for simulating natural and industrial processes with number of particles in order of millions or more. The objective of the current work is to seek a more efficient computational model in terms of memory resources and calculation time.

2. Numerical method

In Lagrangian approaches, the path of each particle is predicted by the integration of its equation of motion resulting from the application of Newton's law of motion. In the point force (or point mass) model, the forces are described as a linear combination of individual contributions of different forces. A rigorously derived equation of motion for small particles in non-uniform flow derived by Maxey and Riley (1983) forms a baseline equation of motion to which other force contributions are added. Neglecting forces arising from non-continuum effects, rotating reference frames, and lift forces, the equation of motion of particles interacting with other particles or walls can be written as:

$$\frac{d\vec{u}_p}{dt} = \vec{f}_{Grav} + \vec{f}_{Drag} + \vec{f}_{Addm} + \vec{f}_{FS} + \vec{f}_{Hist} + \sum \vec{f}_{Contact} \tag{1}$$

where the forces per unit mass on the right-hand-side are due to gravity, drag, added mass, fluid-stress, history, and collision forces, respectively. For collision forces, soft or hard sphere models are commonly used to model particle interactions with other particles or with solid boundary in point force simulations. The soft sphere approach is used in the current work in which the contact force is decomposed into a normal force component and a tangential force component. The normal and tangential force components are modeled using a spring-dashpot-slider system. Details about the soft sphere model can be found in Cundall (1979) and Tsuji et al. (1993). In what follows, force models for solid spherical particles are presented.

Gravity force

Gravity force herein accounts for the weight of the particle (per unit mass of the particle) and is given by;

$$\vec{f}_{Grav} = \vec{g} \tag{2}$$

where g is the gravitational acceleration.

Drag force

The drag force accounts for the quasi-steady viscous dissipation due to skin friction and form drag of the particle in uniform flow. The drag force is often a key element in interaction of dense granular flows. For the current work it will be modeled using the form;

$$\vec{f}_{Drag} = \frac{3C_D\rho_f}{4d_p\rho_p}|\vec{u} - \vec{u}_p|(\vec{u} - \vec{u}_p) \tag{3}$$

where $\vec{u}$ *and* $\vec{u}_p$ is the fluid velocity at the particle location and the particle velocity respectively. ρ_f, ρ_p, C_D and d_p are the fluid density, particle density, the drag coefficient, and the particle diameter. The drag coefficient for a sphere can be expressed as;

$$C_D = \begin{cases} 24\left(1.0 + 0.15Re_p^{0.687}\right)/Re_p, & Re_p \le 1000 \\ 0.44, & Re_p \ge 1000 \end{cases} \tag{4}$$

In which the particle Reynolds number Re_p is calculated as;

$$Re_p = \frac{d_p|\vec{u} - \vec{u}_p|}{\nu_f} \tag{5}$$

where ν_f denotes the fluid kinematic viscosity.

Added mass

The added mass effect accounts for the force required to accelerate/decelerate the fluid surrounding the particle. A general formula derived by Lamb (1945) for an isolated particle in creeping flow limit ($Re_p \ll 1$) is given by;

$$\vec{f}_{Addm} = c_\forall \frac{\rho_f}{\rho_p}\left[\frac{D\vec{u}}{Dt} - \frac{d\vec{u}_p}{dt}\right] \tag{6}$$

Here $c_\forall$ is the added mass coefficient which is equal to 0.5 for a spherical particle, and the derivative Du/Dt is the change in fluid velocity following the particle path. Several studies have shown that the added-mass term predicted by creeping flow and potential flow theory works remarkably well for finite-Reynolds-number flows over a wide range of relative accelerations (Mei et al., 1991; Rivero et al., 1991; Chang and Maxey, 1994, 1995).

Fluid-stress force

The fluid-stress force accounts for the fluid stress-gradients acting across the volume occupied by the particle. The fluid-stress force acting on a particle surrounded by a flow combines the effects of pressure gradients and viscous stresses of the fluid;

$$\vec{f}_{FS} = \frac{1}{\rho_p}\left(-\vec{\nabla}p + \vec{\nabla}\cdot\bar{\bar{\tau}}\right) \tag{7}$$