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Selection of solitary waves in vertically falling liquid films

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ABSTRACT

Two-dimensional solitary waves at the surface of a film flow down a vertical plane are considered. When the system is subjected to inlet white noise, solitary waves are formed after an inception region and interact with each other. Using open-domain simulations of reduced equation models, we investigate numerically their late time process dynamics. Close to the instability threshold, the waves synchronize themselves into bound states. For higher values of the Reynolds number, the separation distance between the waves increases and the synchronization process at work is weaker. Performing statistics, we show that the mean characteristics of the waves correspond to the minimal value of the mean film thickness along the traveling-wave branch of solutions. In this regime, synchronization occurs through the waves tails which is associated with a change of scaling of the waves features. A similar behavior is observed performing simulations in periodic domains: the selected waves maximize the mean flow rate.

1. Introduction

Falling liquid films are widely used in industrial applications due to their interesting properties regarding heat and mass transfer. Interfacial waves intensify transfers by a factor up to two in comparison to the flat film situation (Frisk and Davis, 1972), though the exchange surface undergoes an increment of only a few percent (Portalski and Clegg, 1971). The reason for this paradox lies in the organization of the wavy motion of the film around isolated structures, or solitary waves, interacting one with the other through nearly flat regions of the film. These so-called solitary waves are composed of a main hump, a series of capillary ripples and a flat part called substrate. Whenever the amplitude of these waves is high enough, a recirculation region, i.e. a roll, appears in the main hump (Rohlfs and Scheid, 2015): this mixing is a key element to transfer intensification (Yoshimura et al., 1996). At given flow conditions, the onset of recirculation regions is sensitive to the separation distance between waves and therefore to the frequency (Rastaturin et al., 2006). It is thus essential to determine the preferred wavelength of the natural evolution of the film. This selection process is the subject of the current paper.

The onset of the different regimes of the film dynamics is function of the reduced Reynolds number, denoted δ, and introduced by Shkadov (1977). This parameter combines inertia, viscosity and surface tension and will be defined in Section 2.1. When $\delta < 1.5$, inertia effects are weak and the amplitude of the waves remains small (drag-gravity regime). Selection of the waves in this case has been the object of several works, within the framework of the hydrodynamic bound states theory, and based on various equation models: the weakly nonlinear Kawahara equation (Kawahara and Toh, 1988; Pradas et al., 2012; Tseluiko and Kalliadasis, 2014), the Benney equation (Elphick et al., 1991), the Kapitza–Shkadov model (Chang and Demekhin, 2002) and more recently the Ruyer-Quil–Manneville model (Pradas et al., 2011). A discrete set of preferred wavelengths is observed as waves tend to form bound states through interaction of their tails with the capillary ripples at the front of the next ones. These separation distances correspond to discrete values of stability of the waves with respect to infinitesimal perturbations, and to local minima of the viscous dissipation function along the traveling-wave branch of solitary waves.

When $\delta > 1.5$, the amplitude and speed of the waves increase drastically (drag-inertia regime). In that regime, the natural evolution of the waves show some similarities with the inverse energy cascade observed in 2D turbulence (Kliakhandler, 2000). One can observe successively formation of linear waves, their non-linear saturation, evolution towards solitary waves through quasi periodic or subharmonic secondary instabilities, wave merging and increasing of the separation distance between solitary waves. The value

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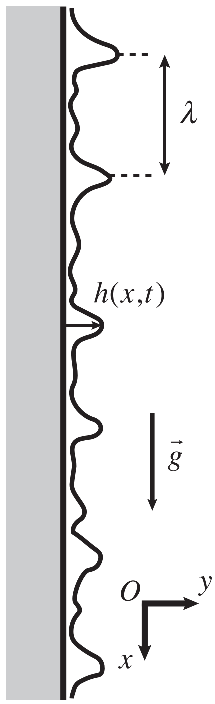

Fig. 1. Sketch of the problem setup.

to which this separation distance converges is still an open question in the literature. It has been addressed only by Chang and Demekhin (2002) but using the Kapitza–Shkadov model and making strong assumptions on the details of the dynamics. Other studies are available in the turbulent case on the coarsening of the so-called roll waves (Balmforth and Mandre, 2004).

Eventually, let us introduce the concept of optimal wave which is the main motivation of this study. It is due to Trifonov (2014) and corresponds to a minimum of the mean height along the traveling-wave branch of solitary waves. Short and long optimal waves are distinguished, depending on the value of the associated separation distance. We will show in this paper by means of time simulations of the Ruyer-Quil–Manneville model that the long-time dynamics of the film converges to these solutions. Links will be made with results from the linear stability analysis, and with the argument of minimal viscous dissipation.

The paper is organized as follows. Section 2 presents the numerical tools. In Section 3, we revisit the literature results in the drag-gravity regime ($\delta > 1.5$). Section 4 presents the results obtained in the drag-inertia regime. Conclusions are given in Section 5.

2. Numerical tools

2.1. Notations

We consider a liquid film falling down a vertical plane (Fig. 1). The flow is assumed to be Newtonian with constant physical properties (surface tension σ, viscosity μ, density ρ). We denote by $\nu = \mu/\rho$ the kinematic viscosity and g the acceleration of gravity. The current study is carried out in the 2D case, that is only by considering spanwise invariant waves with x being the coordinate in the direction of the flow and y the coordinate oriented in the normal direction to the plane. u and v refer to the velocity components in the two directions x and y.

Two length scales can be defined balancing viscosity, acceleration of gravity and surface tension: the capillary length $l_c = \sqrt{\sigma/\rho g}$ and the viscous-gravity length $l_\nu = \nu^{2/3} g^{-1/3}$. Adding the flat film thickness $\bar{h}_N$ (Nusselt thickness) allows to get a first set of non-dimensional parameters. It is composed of the Kapitza number $\Gamma = (l_c/l_\nu)^2$ and the Reynolds number $Re = \bar{q}_N/\nu = h_N^3/3$ built from the flow rate $\bar{q}_N$ or the non-dimensional Nusselt thickness $h_N = \bar{h}_N/l_\nu$.

The Kapitza number takes rather high values in practice which is not convenient from a numerical point of view. This led Shkadov (Shkadov, 1977) to introduce a specific non-dimensionalization, compressing the x coordinate by a factor κ with respect to the y direction. This factor is adjusted to balance the gravity force and the capillary pressure gradient, i.e. $\kappa = (l_c/\bar{h}_N)^{2/3}$. The Shkadov scaling yields two non-dimensional numbers: the reduced Reynolds number $\delta = 3Re/\kappa$ and a viscous dispersion parameter $\eta = 1/\kappa^2$. It sets the coefficient of the capillary pressure term to one and makes explicit the balance of all forces in the equations (gravity, viscosity, surface tension and inertia).

2.2. WRIBL model

Hereinafter we will mimic the dynamics of the film based on the solutions to a reduced set of equations involving only two variables: the film thickness $h(x, t)$ and the local liquid flow rate $q(x, t) = \int_0^h u dy$. This model has been validated through comparisons to DNS and experiments (Kalliadasis et al., 2012) showing excellent agreement for the parameter range of interest. It allows to get access to the essence of the flow dynamics at reasonable computational costs. Let us briefly outline its derivation procedure starting from the governing equations.

First, the long-wavelength nature of the instability enables to invoke a separation of scale between the x and y coordinates. In practice, this is performed by introducing a small parameter $\epsilon = \partial_{x,t}$ and ordering terms into powers of ϵ. Then, the classical boundary-layer approximation is followed: pressure is computed after integration of the y-momentum balance where $O(\epsilon^2)$ inertial terms have been dropped out. Depth-averaging of the x momentum equation yields an evolution equation for the flow rate consistent up to $O(\epsilon^2)$. Let us mention that using specific weights through this latter procedure allows considerable algebraic simplification (weighted-residual technique, see Kalliadasis et al. (2012) for more details). The so-obtained Weighted Residual Integral Boundary Layer model is composed of a (exact) depth-averaged mass-conservation equation and a momentum-conservation equation. It is written as:

$$\partial_t h + \partial_x q = 0, \tag{1a}$$

$$\delta \partial_t q = \frac{5}{6} h - \frac{5}{2} \frac{q}{h^2} + \delta \left[\frac{9}{7} \frac{q^2}{h^2} \partial_x h - \frac{17}{7} \frac{q}{h} \partial_x q \right] + \frac{5}{6} h \partial_{xxx} h$$
$$+ \eta \left[4 \frac{q}{h^2} (\partial_x h)^2 - \frac{9}{2h} \partial_x q \partial_x h - 6 \frac{q}{h} \partial_{xx} h + \frac{9}{2} \partial_{xx} q \right]. \tag{1b}$$

As already underlined, this model is consistent up to $O(\epsilon^2)$ with the long-wave expansion and thus adequately accounts for the second-order viscous effects, i.e. the elongational viscous terms (last row of (1b)), which are omitted in most low-dimensional models, for instance the classical Kapitza–Shkadov model (Shkadov, 1967) as the viscous dispersion parameter η is generally small. Yet, the linear stability of the Nusselt base flow is significantly affected by the presence of these terms as they decrease the speed of kinematic waves. We recall here that the imbalance of the kinematic-wave speed with the flow speed at the free surface is the key ingredient for the onset of the instability as pointed by Smith (1990) and Kalliadasis et al. (2012) within the Whitham wave-hierarchy framework (Whitham, 1974).