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A power flow solvability identification and calculation algorithm

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Abstract

This paper presents a continuation and optimization based algorithm to detect power flow unsolvability. In addition, the algorithm obtains the power flow solution, if it exists, no matter how ill-conditioned the power system is. The proposed algorithm is based on the parameterization of the distance from the starting point to the real power flow to be solved, using a convergence margin. The performance of the algorithm is illustrated considering an highly loaded scenario of the operation of the Spanish power system.

Keywords: Power flow solvability; Ill-conditioned systems; Convergence margin; Continuation; Optimization

1. Introduction

Power flow is the key tool in power system planning and operation. A power flow program provides the steady-state solution of a power system scenario. A scenario is defined by the network topology, the loads, the active power generation and the generator voltages. The power flow solution of a scenario built from scratch is not always easy to reach. Therefore, it is of great interest the development of a tool able to determine the solvability of the power flow solution of a scenario and to provide such solution, no matter how close to the voltage collapse is the operating point.

The power flow problem is formulated as a set of non-linear equations that includes parabolic and trigonometric functions. Due to the non-linearity of the power flow equations, all the power flow methods developed in the literature have been designed as iterative processes.

The first power flow method, now known as Gauss–Seidel (GS), was developed during the 1950s [1]. However, the most commonly method used for power flow calculation is the Newton–Rhapson (NR) method, based on the successive solutions of the linear approximation of the power flow equations. The NR method exhibits better convergence conditions than the GS method. The NR method became solvable because of the development of high efficient sparse matrix techniques for direct solution of linear equations systems [2].

The conventional power flow methods are known to have difficulties to converge in case of ill-conditioned systems. In such cases, the iterative process may either oscillate or diverge. From a physical point of view, the non convergence of the power flow equations of a power system can be due to a number of features of the network, such as the generator voltages, the load of the system, the generation–demand imbalance between the power system areas, etc. However, convergence problems are due to the nature of power flow system equations. The nature of power flow system equations is quadratic (voltage magnitudes) and trigonometric (voltage angles). Therefore, as a consequence of this non-linear nature, the power flow can be solvable or unsolvable. A solvable power flow has two different real solutions for the state variables (voltages both magnitude and angle). However, an unsolvable power flow has two different complex solutions for the state variables, which are non admissible from a technical point of view since voltages magnitude and argument are considered real variables.

It is important to remark the difference between *solvability* and *feasibility* [3,4]. A feasible power flow not only is solvable, but also all system variables (e.g. bus voltages, line

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flows) are within their limits, whereas only reactive power generation limits are considered in power flow solvability analysis. Usually it is possible to operate the system (at least for a while) in the infeasible region, whereas any attempt to operate it in the unsolvable region there would probably result in system instability and voltage collapse [5,6].

Solvable and unsolvable region in parameter space are separated by a boundary. In this boundary, a saddle-node bifurcation exists, thus the Jacobian matrix of the power flow equations is singular. If the power system is close to the boundary, the Jacobian matrix is also close to singularity and the system is ill-conditioned (high sensitivity from input data to output results). The ill-conditioning of a power system make the power flow algorithms convergence worse. Hence, the solution of a solvable power flow problem could either oscillate or diverge if the system is ill-conditioned.

Thus, the non convergence of a power flow can be due to the proximity to voltage collapse or due to an unsolvable operating point.

To overcome the non convergence of the standard NR method, a number of developments have been undertaken. Clements et al. incorporate in [7] a deceleration factor to the state variables updating vector, controlled by the mismatches evolution during the convergence process. Later, this step size control has been optimized, not only improving the convergence, but also using the deceleration factor as a power flow solvability indicator. Braz et al. presents in [8] a critical evaluation of three methods of step size optimization based polar coordinates power flow methods developed in [9–11], whereas step size optimization based rectangular coordinates power flow are developed in [12,13].

However, these algorithms only identify when a case is unsolvable, but do not provide any unsolvability measure. Overbye presents in [3] an algorithm that not only identifies unsolvable cases, but also provides a measure of how unsolvable the case is. This unsolvability measure is based in the minimum Euclidean distance between the operating point, and the closest solvable operating point in the parameter space.

This paper presents a continuation and optimization based algorithm to detect power flow unsolvability. In addition, the algorithm obtains the power flow solution, if it exists, no matter how ill-conditioned the power system is, thanks to continuation techniques robustness [12,14]. The proposed algorithm is based on the parameterization of the distance from the starting point to the specified power flow to be solved, using a convergence margin. This convergence margin is bounded between zero and one. The algorithm maximizes the convergence margin subject to the augmented power flow equations. If the convergence margin maximum is equal to one the power flow solution is obtained, whereas if the convergence margin maximum is less than one the power flow is unsolvable. To solve the optimization problem, both continuation [14,15] and optimization [16,17] techniques are employed. To combine continuation and optimization techniques, a least-squares minimization based Lagrange multipliers estimation is introduced [18], providing advantages of both techniques: continuation robustness [12,14] and optimization Lagrange multipliers, which can be used as useful sensitivities [19,20] for post-optimization parameterization algorithms [21,22].

The process starts considering an initial state variables vector that corresponds to a initial power bus injections different from the specified power bus injections. This initial state variables vector produces an unreal reactive power generation, producing unreal reactive power generation limits violations. To avoid unnecessary unreal reactive power generation limits fixation, the full power flow convergence process has been separated into two mainly steps: convergence *without considering* reactive power generation limits and convergence *considering* reactive power generation limits.

The paper is organized as follows: Section 2 provides a generic formulation of the continuation–optimization algorithm. Section 3 applies the algorithm presented in Section 2 to the power flow, considering the reactive power generation limits. Section 4 illustrates the performance of the proposed algorithm applied to a highly loaded hourly scenario of the operation of the Spanish power system. Finally, Section 5 contains the conclusions of the paper.

2. A mixed continuation–optimization approach to the solution of non-linear equations

2.1. Overview of the algorithm

Let us start formulating a set of non-linear equations:

$$\mathbf{g}(\mathbf{x}, \lambda) = \mathbf{0} \tag{1}$$

where $\mathbf{x}$ is the system state variables, $\mathbf{g}(\mathbf{x}, \lambda)$ the augmented system state equation, and λ is the convergence margin.

Two different augmented system state equations $\mathbf{g}(\mathbf{x}, \lambda)$ formulations are used in this paper. One formulation is employed for convergence *without considering* reactive power generation limits, whereas convergence *considering* reactive power generation limits uses a different formulation. Both augmented system state equations $\mathbf{g}(\mathbf{x}, \lambda)$ formulations used in this paper will be detailed in Section 3.

The convergence margin λ is bounded by zero and one. Thus:

$$\lambda \in [0, 1] \tag{2}$$

The problem consists of maximizing the convergence margin λ subject to the set of non-linear Eq. (1) and the upper bound of the convergence margin λ:

$$\begin{aligned} \max \quad & \lambda \\ \text{s.t.} \quad & \mathbf{g}(\mathbf{x}, \lambda) = \mathbf{0} \\ & \lambda - 1 \leq 0 \end{aligned} \tag{3}$$