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A simplified two-phase flow model using a quasi-equilibrium momentum balance

Ulf Jakob F. Aarsnes[a,1,*], Adrian Ambrus[b], Florent Di Meglio[c], Ali Karimi Vajargah[d], Ole Morten Aamo[a], Eric van Oort[d]

[a] Department of Engineering Cybernetics, NTNU, Trondheim, Norway
[b] Department of Mechanical Engineering, University of Texas at Austin, United States
[c] Centre Automatique et Systmes, MINES ParisTech, France
[d] Department of Petroleum and Geosystems Engineering, University of Texas at Austin, United States

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ABSTRACT

We propose a simple model of two-phase gas–liquid flow by imposing a quasi-equilibrium on the mixture momentum balance of the classical transient drift-flux model. This reduces the model to a single hyperbolic PDE, describing the void wave, coupled with two static relations giving the void wave velocity from the now static momentum balance. Exploiting this, the new model uses a single distributed state, the void fraction, and with a suggested approximation of the two remaining static relations, all closure relations are given explicitly in, or as quadrature of functions of, the void fraction and exogenous variables. This makes model implementation, simulation and analysis very fast, simple and robust. Consequently, the proposed model is well-suited for model-based control and estimation applications concerning two-phase gas–liquid flow.

Introduction

Multi-phase flow simulation models have evolved significantly over the last couple of decades. With the increase in computational power and sophistication of numerical schemes, simulating two-phase pipe flow no longer suffers the same limitations on computational size, and state of the art high-fidelity models such as OLGA (Bendiksen et al., 1991) and LedaFlow (Danielson et al., 2011) typically run many times faster than real-time on a standard desktop computer.

Before this development, however, significant efforts were devoted to obtaining simplifications of multi-phase flow models which could ease implementation and increase their simulation speed. The Drift Flux Model (DFM) (Ishii, 1977) was first proposed by Zuber and Findlay (1965) as a correlation for predicting steady-state void-fraction profiles and later used in transient representations of two-phase flow (Pauchon and Dhulesia, 1994). In this form it is a simplification of the transient two-fluid model obtained by relaxing (i.e. imposing immediate steady-state on (Flåtten and Lund, 2011)) the dynamic momentum equation of each phase, replacing them with a mixture momentum equation and a static relation typically called a *slip law*.

Further simplification can be achieved by using a similar approach to other parts of the dynamics deemed insignificant for the application at hand. Specifically, by imposing steady state on the momentum balance, the pressure wave dynamics are neglected, yielding so-called “No Pressure Wave” (NPW) models or “Reduced DFMs”. This simplification is motivated by applications for which slow gas propagation dynamics are more critical than fast pressure wave propagation. Furthermore, it has been shown that the validity of the drift-flux models representation of the fast pressure dynamics is imprecise in many scenarios due to the relaxations involved in obtaining the DFM from the full formulation of

* Corresponding author.
E-mail address: ulfjakob@gmail.com, ulf.jakob.aarsnes@itk.ntnu.no (U.J.F. Aarsnes).
[1] Now with IRIS, Stavanger, Norway.

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Baer and Nunziato (1986), which lowers the sonic velocity (Flåtten and Lund, 2011; Linga, Submitted). Thus, if the “medium” complexity DFM representation of the pressure waves is imprecise, the argument can be made that they could be discarded.

This approach was used by Taitel et al. (1989) where the resulting model was described by a single transient PDE of the liquid continuity, obtained by assuming incompressible liquid, and a set of steady-state relations. The resulting model was further investigated by Minami and Shoham (1994) where it was found to be amenable for certain scenarios. The approach was expanded upon by Taitel and Barnea (1997), where the assumption of incompressible liquid was dropped, yielding two transient equations. A similar model was investigated by Masella et al. (1998), here called the “No Pressure Wave” (NPW) model. More recent additions to the literature on models using quasi-equilibrium momentum balance include (Choi et al., 2013; Aarsnes et al., 2015; Ambrus et al., 2015).

Interestingly, many of these recent studies have not been motivated by the desire to reduce computational complexity. Rather, the advent of computerized automation and optimization in the oil and gas industry has created new applications for various forms of simplified models, causing renewed interest in these models.

Application

Modern advances in the theory of dynamic systems have the potential of improving robustness and performance in the monitoring, optimization and control of dynamic processes which can be described by an amenable mathematical model. By intelligently combining predictions from the mathematical model with information from multiple sensors one can estimate unmeasured quantities, optimize automatic control procedures, predict future behavior, and plan countermeasures for unwanted incidents. Such design techniques, often referred to as model-based estimation and control (Åström and Murray, 2010; Anderson and Moore, 1990), require a mathematical model with the proper balance between complexity and fidelity, i.e. the complexity must be limited to facilitate the use of established mathematical analysis and design techniques, while the qualitative response of the process is retained.

Models that achieve such balance between complexity and fidelity are sometimes referred to as fit-for-purpose models. Obtaining such models often proves difficult for gas–liquid two-phase dynamics due to the significant complexity and distributed nature of multi-phase pipe flow (Aarsnes et al., 2014; 2016).

If the appropriate model can be developed, however, it could see a wide range of uses in model-based control and estimation applications where two-phase pipe flow is encountered, such as underbalanced drilling of oil and gas wells (Pedersen et al., 2015), well control (both in conventional and Managed Pressure Drilling) (Carlsen et al., 2008), riser gas handling (Hauge et al., 2015), hydrocarbon production monitoring (Bloemen et al., 2006; Teixeira et al., 2014) and mitigating severe slugging during hydrocarbon production (Eikrem et al., 2008; Esmaeil and Skogestad, 2011; Di Meglio et al., 2010a).

The drift flux model

A popular model for representing one-dimensional two-phase flow dynamics in drilling and production at an acceptable fidelity is the classical three-state transient Drift Flux Model (DFM), see e.g. (Lage and Time, 2000; Fjelde et al., 2003; Aarsnes et al., 2014).

For certain boundary conditions, the existence of solutions has been proven (Evje and Wen, 2013; 2015), and it is well known that the DFM is, in most practical situations, hyperbolic, with three (two fast and one slow) characteristic velocities (Di Meglio, 2011). The two fast characteristics represent the fast pressure dynamics in the pipe, while the slow characteristic velocity is associated with the transport of matter, also sometimes referred to as the void wave (Lorentzen and Fjelde, 2005; Masella et al., 1998).

In this section we restate the classical equations of the transient drift-flux model and then cast the system in canonical form using the eigenvectors of the transport matrix, which poses the model as a single Riemann invariant governing the propagation of the void wave, coupled to the pressure dynamics, given by two PDEs, through the gas velocity. We then show how the approximation employed by e.g. (Masella et al., 1998; Choi et al., 2013), using a static relation in place of a dynamic momentum balance, is related to relaxing both of the two PDEs describing the pressure dynamics. Both approaches lead to a mixed hyperbolic/parabolic system with a single hyperbolic PDE with a finite eigenvalue.

The drift flux model equations

We start the development of the proposed two-phase model from the classical Drift Flux Model (DFM) formulation, described by the following equations (Gavrilyuk and Fabre, 1996; Evje and Wen, 2015):

$$\frac{\partial(\alpha_L\rho_L)}{\partial t}+\frac{\partial(\alpha_L\rho_L v_L)}{\partial x}=\Gamma_L, \tag{1}$$

$$\frac{\partial(\alpha_G\rho_G)}{\partial t}+\frac{\partial(\alpha_G\rho_G v_G)}{\partial x}=\Gamma_G, \tag{2}$$

$$\frac{\partial(\alpha_L\rho_L v_L+\alpha_G\rho_G v_G)}{\partial t}+\frac{\partial(P+\alpha_G\rho_G v_G^2+\alpha_L\rho_L v_L^2)}{\partial x}=S, \tag{3}$$

where the independent variables t, x represent time and position along the pipe, respectively, and the momentum source term, S is typically given as

$$S=-\rho_m g\sin\theta(x)-\frac{2f\rho_m v_m|v_m|}{D} \tag{4}$$

with the mixture relations

$$\rho_m=\alpha_G\rho_G+\alpha_L\rho_L,\quad v_m=\alpha_G v_G+\alpha_L v_L, \tag{5}$$

and where α_i, v_i, ρ_i, Γ_i denote the volume fraction, velocity, density and mass source term, respectively, of phase $i=G,L$ (gas or liquid). Finally, f is the friction coefficient, D the hydraulic diameter, g is the acceleration of gravity and θ is the pipe inclination angle (relative to the horizontal). For the remainder of this section we will assume $\Gamma_L=\Gamma_G=0$.

Eqs. (1) –(2) represent the mass balance for the liquid and gas phases, while (3) is the conservation of momentum for the gas–liquid mixture.

The following closure relations are needed to complete the system:

$$\alpha_L+\alpha_G=1,\quad P=c_G^2(T)\rho_G, \tag{6}$$

where P is the pressure, and $c_G(T)$ is the speed of sound in the gas as a function of the temperature T, while the liquid density is assumed constant. Finally the slip law

$$v_G=\frac{v_m}{1-\alpha_L^*}+v_\infty=C_0v_m+v_\infty \tag{7}$$

where the profile parameter $\alpha_L{}^*\in[0,\,1)$, usually given as the distribution parameter $C_0=1/(1-\alpha_L^*)$, and drift parameter $v_\infty\geq 0$ determine the relative velocity between the phases. These parameters typically depend on factors such as superficial velocities and inclination (Shi et al., 2005). Multiple correlations for obtaining α_L^*, v_∞ exist in the literature, see e.g. (Zuber and Findlay, 1965; Bhagwat and Ghajar, 2014; Choi et al., 2012).