International Journal of Multiphase Flow 67 (2014) 10–24

Contents lists available at ScienceDirect

International Journal of Multiphase Flow

journal homepage: www.elsevier.com/locate/ijmulflow

Three-dimensional deformation of a spherical droplet in a square duct flow at moderate reynolds numbers

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ARTICLE INFO

Article history:
Received 11 February 2014
Received in revised form 21 July 2014
Accepted 23 July 2014
Available online 1 August 2014

Keywords:
Droplet deformation
Three-dimensional shear flow
Square duct
Lattice Boltzmann Method
GPU
Confined flows

ABSTRACT

In this work, we study the dynamics of a three-dimensional neutrally buoyant droplet as it translates and deforms in a square-duct. An emphasis is placed on understanding the transient deformation history. We study the effect of the Reynolds number, capillary number, and droplet to carrier fluid viscosity ratio on deformation. In the low Reynolds number limit, we expect the droplet deformation to be independent of Reynolds number. We examine moderate Reynolds numbers in the range of 10–100 and show that inertia plays a significant role in droplet deformation both in the magnitude of steady-state as well as transient deformations. For Reynolds numbers greater than 25, we observe non-monotonic deformation histories. At higher Reynolds numbers we also observe cavity regions forming at the trailing edge of the droplets. These cavity regions appear to be unstable due to their relatively high interfacial curvature. Interfacial tension therefore plays a significant role at the trailing edge to return the cavity region to a more stable convex shape. We study droplet deformation for moderate capillary numbers in the range of 0.10–0.25 and observe a strong variation in deformation magnitude with capillary number. However, the capillary number does not appear to influence the qualitative transient deformation behavior. We also explore the effect of droplet to carrier fluid viscosity ratio on droplet deformation and observe a more complicated relationship. For small viscosity ratios, the deformation histories are non-monotonic and the steady-state deformation increases as the viscosity ratio is increased. In this regime, a cavity forms at the droplet trailing during early times. The formation of a trailing edge cavity is considerably decreased and vanishes for large droplet viscosity. For large drop viscosity, the system is considerably damped and the deformation histories are monotonic for the period of time investigated in this study.

1. Introduction

The deformation of an immiscible spherical droplet in a shear flow of another fluid has been a fundamental mechanics problem for approximately 80 years since the pioneering experimental and theoretical study of Taylor (1934). Taylor's two innovative experiments involving counter-rotating rollers and film strips created different shear patterns on a spherical droplet that deformed to elongated ellipsoidal shapes of different aspect ratios. The deformation parameter defined as $D = (L - B)/(L + B)$ was shown to be linear over a range of the F parameter ($F \equiv 2C\mu a/T$, where C, μ, a, and T represent respectively the hyperbolic velocity constant, surrounding fluid viscosity, droplet size and tangential stress), after which the theory and experiments differed due to large shear stress. Following Taylor's work, a number of studies on droplet deformation and motion in shear flows have appeared. Many of these have been concentrated on two specific flows: Poiseuille flow in a circular pipe, and shear flow between parallel plates. A few studies have been concerned with the more complex shear flow in a square duct. A review of previous observations is given below.

An extensive review of work done prior to 1994 on drop deformation and breakup has been presented in Stone (1994). The review discusses work done on drop deformation in straining flow, breakup due to large shear rate in linear flow, and satellite droplet formation due to the pinching of a highly elongated cylindrical drop. A review, up to 2010, of work done on droplet deformation in Poiseuille flow in circular and square ducts has been presented by Guido and Preziosi (2010). The review included the effect of non-dimensional parameters such as the droplet to carrier fluid dynamic viscosity ratio ($\lambda = \mu_d/\mu_c$), confinement ratio ($a = r/R$, where r = drop radius and R = tube radius), capillary number ($Ca = \mu_d u_b/\sigma$) and Reynolds number ($Re = \rho_d u_b d/\mu_d$, where

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http://dx.doi.org/10.1016/j.ijmultiphaseflow.2014.07.010
0301-9322/

$d =$ droplet diameter and $u_b =$ bulk velocity) on the drop deformation, drop velocity, cross-stream or lateral migration and additional pressure drop due to the presence of the drop. We review below some of the works covered in these reviews and recent research since then.

The motion of small droplets between two stationary plates has been studied by Shapira and Haber (1988). They used the method of reflections to study the pressure distribution along with the drag force exerted on a droplet. The pressure distribution along the plate was found to be anti-symmetric in the stream-wise direction and attains a peak at small values of stream-wise distance. The drag force on the droplet depends on the wall-normal distance to the plates and is shown to be symmetric with respect to the channel half-height with the minimum value at the channel mid-plane. Shapira and Haber (1990) later used the same method of analysis for a droplet in a Couette flow and found the drag coefficient to be anti-symmetric about the channel centerline for different droplet positions. The droplet positioned at the channel center-plane would experience no drag. Droplets near the stationary wall would experience a maximum positive drag while droplets near the moving wall would experience a maximum negative drag (thrust).

The lateral migration of a deformable droplet in two-dimensional Poiseuille flow at moderate Reynolds numbers has been studied by Mortazavi and Tryggvason (2000). They studied the effects of the drop size, density and viscosity ratios, as well as Reynolds and Weber numbers on droplet equilibrium lateral position. For ($Re \approx 1$), lower viscosity ratio ($\lambda \approx 0.125$) drops move toward the center of the channel, whereas more viscous drop ($\lambda \approx 1$) moved away from the center. For $Re \in (5 - 25)$, the equilibrium lateral position was approximately half-way between the channel wall and channel half-height. At larger Reynolds numbers ($Re \in [37 - 50]$), the lateral position became oscillatory although the oscillations were decaying for this range of Reynolds numbers. Griggs et al. (2007) performed three-dimensional simulations of droplet deformation suspended in Poisseuile flow between parallel plates using a boundary integral method. The droplet velocity was smaller than the undisturbed center-line velocity for all capillary numbers ($Ca \in [0.3 - 0.7]$) and viscosity ratios ($\lambda \in [0.5 - 25]$). The difference in drop and centerline velocities decreased as the capillary number increased and the viscosity ratio decreased. The lateral migration rate showed non-monotonic behavior with viscosity ratio. Droplets beginning away from the channel center-plane experienced lateral migration and came to an equilibrium position near the channel half-height for all capillary numbers and viscosity ratios considered. While the migration rate was monotonic with respect to the capillary number (higher capillary number droplets reached the channel center-line in less time), the migration rate was non-monotonic with respect to the viscosity ratio; however, the general trend was that less viscous droplets reached the channel center-plane quicker than the more viscous droplets.

Sibillo et al. (2006) studied deformation of a single droplet in shear flow between parallel plates. For a low confinement droplet $r/H = 0.07$ (where r is the droplet radius and H is channel width), and moderate capillary number $Ca = 0.4$ the droplet continuously deformed from a spherical configuration to a steady state highly elongated ellipsoidal shape. When the shear was removed, the drop relaxed toward an equilibrium spherical shape. In contrast, for a large confinement ratio of $r/H = 0.5$ and the same capillary number, the drop deformation history was non-monotonic. In addition, the droplet's steady state shape was sigmoidal. The confined droplet's steady state deformation exceeded the relatively unconfined droplet's deformation by approximately a factor of 1.7. Their results also suggest that confinement can stabilize droplets which could otherwise break-up at a higher deformation for a given capillary number. Janssen and Anderson (2007) investigated drop deformation in a shear flow between two parallel plates using a boundary integral method. For $Ca = 0.2$, a non-monotonic deformation history was observed for confinement ratios approximately 0.67 and greater. Similar to the observations of Sibillo et al. (2006), these authors also observed that highly confined droplets undergoing shear exhibited sigmoidal steady state shapes. Janssen and Anderson (2008) performed a similar study for non-unity viscosity ratio droplets. For $Ca = 0.3$, viscosity ratio (λ) $= 10$, and confinement ratio $(2r/H) \in [0 - 0.83]$, the deformation was oscillatory i.e. first there was rapid elongation, followed by slow contraction and subsequent constant deformation along the longitudinal direction. However, the oscillations appeared to decrease with increasing confinement. Lower viscosity ratio drops ($\lambda \leqslant 1$) at the same capillary number exhibited less sensitivity of their deformation on changing confinement. In addition, increased confinement was observed to reduce rotation of the droplet's major axis. This kept the droplet more aligned with the flow direction (compared with an unconfined droplet) which therefore led to greater deformation.

Vananroye et al. (2008) performed experiments and numerical simulations of droplet deformation in shear flow. Similar to Sibillo et al. (2006) and Janssen and Anderson (2008) they also observed that increase in the confinement ratio increased droplet deformation and aligned the droplet's major axis toward the flow direction. They also observed oscillatory behavior in the deformation history for highly confined droplets while relatively unconfined droplets exhibited monotonic deformation histories.

Khan and Wang (2010) used a three-dimensional spectral boundary element method to study droplet deformation in a shear flow. Similar to previous observations, they observed that more confined droplets experienced greater deformation. Deformation is observed to increase with increasing capillary number and decreasing wall-normal distance (for the case when one wall is close to the droplet and the second wall is far away from the droplet). Deformation was observed to increase monotonically with viscosity ratio for ($\lambda \in [0.1, 1.5]$) while deformation decreased monotonically (for $\lambda \in [1.5, 10]$). Wang and Dimitrakopoulos (2011) also used a three-dimensional spectral boundary element method to simulate low Reynolds number droplet motion and deformation in a square micro-channel. They observed monotonically increasing steady-state deformation as the viscosity ratio increased for $\lambda \in [0.5, 10]$. Deformation was also observed to increase with confinement; confinement had a greater influence on deformation for larger droplets. The droplet stream-wise velocity was observed to increase as the capillary number increased, while the droplet velocity decreased as the viscosity ratio increased. This suggests a droplet's deformation and its stream-wise velocity are negatively correlated. Wang and Dimitrakopoulos (2011) also compared the deformation of a droplet in a square duct to the deformation in a circular tube and observed higher deformation in the tube for a fixed confinement ratio. This observation is consistent with those given by Kuriakose and Dimitrakopoulos (2011) who did an extensive comparison of deformation of an elastic capsule in both a square duct and a circular tube. These studies were limited to low Reynolds numbers.

Nourbakhsh et al. (2011) simulated an array of droplets in plane Poiseuille flow. The droplet concentration peaks about halfway between the wall and channel center-plane for droplets experiencing small deformation. In contrast, highly deformable drops migrate towards the middle of the channel and exhibit a higher concentration at the channel center-plane. That is, for low capillary number ($Ca = 0.05$) and relatively large Reynolds number $O(10)$, drops concentrate in the wall region. But for higher capillary numbers, the droplet concentration is highest near the channel center-plane.