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# Shifting and breakup instabilities of squeezed elliptic jets

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ABSTRACT

Liquid jets are inherently unstable and breakup into drops due to capillary instability driven by surface tension. Elliptic jets have been studied and reported to be more unstable than circular jets. The reason for their increased instability has been often ascribed to the axis-switching of elliptic jets. We theoretically elucidate, for the first time, that even in the absence of axis-switching, an elliptic jet exhibits greater instability compared to circular jets solely due to interfacial curvature effects. For this purpose, we analyse the stability of a jet with elliptic cross-section subject to a non-uniform ambient pressure. This azimuthally dependent pressure distribution in the ambient fluid helps to prescribe a static base state for the elliptic jet and maintain the azimuthally varying curvature of the interface. Physically, this mathematical treatment implies that the jet is squeezed into an elliptic configuration by a non-uniform pressure distribution in the passive ambient fluid and therefore cannot exhibit axis-switching. The jet is assumed to be incompressible and inviscid. The stability of such a jet is analyzed using Rayleigh's Work Principle (RWP) and linear stability theory. It is also shown that in addition to capillary breakup, analogous to that of classical circular jets, squeezed elliptic jets are found to exhibit a shifting mode instability, in which the entire jet translates parallel to its major axis. Both capillary breakup and shifting modes are rendered more unstable by an increase in ellipticity. For low ellipticity, the breakup mode exhibits higher growth rates as compared to the shifting mode. We show that there exists a critical ellipticity beyond which the shifting mode starts to dominate the capillary breakup mode. Predictions from both RWP and linear stability theory converge to Rayleigh's classic results for a circular cylindrical jet.

## Introduction

Cylindrical jets of liquid are unstable and under the action of surface tension they break up into drops. This is a consequence of the system wanting to evolve to a state with lower surface energy by minimizing the surface area. This instability is also called capillary instability and was theoretically analyzed for the first time by Rayleigh (1878) for an inviscid stationary circular jet. Several experimental and theoretical studies have thereafter validated and improved upon Rayleigh's theory (Hoeve et al., 2010; Chandrasekhar, 1961; Tomotika, 1935; Weber, 1931; Keller et al., 1973; Meister et al., 1967). However, all of these studies were limited to jets of circular cross section. A liquid jet emanating from an elliptic orifice into an ambient fluid of uniform pressure (like atmosphere) has an elliptic configuration near the orifice. The stability of such a jet has been studied recently (Crighton, 1973; Husain and Hussain, 1983; Bechtel, 1989; Kasyap et al., 2009; Dolatabadi and Amini, 2011). Apart from the academic interest for its fascinating physics, elliptic jets have found applications in several areas of science and engineering. They are found to provide better mixing than circular jets (Husain and Hussain, 1983; Gutmark and Grinstein, 1999) and have applications in spray painting, liquid fuel injections in rocket engines (McHale et al., 1971) and jet noise suppression (Crighton, 1973).

As a liquid jet flows downstream from an elliptic orifice, it is found to exhibit a flow wherein the major and minor axes switch periodically (Amini and Dolatabadi, 2012). Bechtel had studied this phenomenon theoretically, illustrating the effects of surface tension, viscosity, inertia and gravity on axis-switching (Bechtel, 1989). Axis-switching in non-Newtonian jets has also been studied by Bechtel et al. (1988a, 1988b). This axis-switching can be seen as a consequence of the jet constrained to satisfy the normal stress balance at the interface. This stress balance dictates that in the presence of uniform pressure ambient fluid, the equilibrium configuration of the interface is the one with uniform curvature – a circular cross-section. Thus, the jet – that emanated being initially elliptic – is driven towards acquiring a circular cross-section as it flows downstream. However, as the jet approaches the circular equilibrium, inertia of the fluid drives it past its circular equilibrium resulting in periodic switching of major and minor

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axes. It is important to note here that it is the uniform ambient pressure that results in the elliptic jet exhibiting axis-switching. It has been reported that elliptic jets are more unstable as compared to circular jets (Dolatabadi and Amini, 2011; Amini and Dolatabadi, 2011; Kasyap et al., 2009; Amini and Dolatabadi, 2012) and axis-switching is attributed to be the cause of its increased destabilization (Kasyap et al., 2009). Thus, for a jet exiting an elliptic orifice, factors affecting its stability are twofold: (a) elliptic cylindrical geometry of the interface resulting in capillary instability solely due to its curvature (b) departure of elliptic cross-section from the equilibrium circular configuration (a circular cross-section is the equilibrium configuration of the interface in a uniform pressure ambient fluid) resulting in axis-switching.

In this work, we address the fundamental question: *Will an elliptic jet be as stable as a circular jet of equal mass, if it hadn't exhibited axis-switching?* For this purpose, we decouple the effect of departure of an elliptic jet from its circular equilibrium from the effect of interface geometry and focus on how the interface geometry solely affects the stability of the jet. The tendency of an elliptic jet to relax to the equilibrium circular cross-section is eliminated by requiring that the ambient pressure be azimuthally non-uniform in such a way that the elliptic interface itself is the new equilibrium configuration. This essentially requires that the ambient fluid have a higher pressure about the jet's minor axis compared to that about its major axis. This results in "squeezing" of the jet which helps it to retain the elliptic configuration of the interface. In such a case, stability of the jet is affected by its elliptic interface alone. We study the stability of such a jet using two classical methods: Rayleigh's Work Principle (RWP) and linear stability theory. Rayleigh (1878) identified the opposing effects of axial and radial curvatures of a perturbed circular jet as the key mechanism for the selection of a critical wavelength of capillary instability. The elliptic configuration of the squeezed jet alters this competition and affects the growth rates of the disturbance and the resulting drop size. The extension of RWP is carried out for two specific classes of perturbations: (i) those which preserve the focal length, (ii) those which preserve the aspect ratio. In addition, using temporal linear stability analysis we find that for an elliptic jet, a new shifting mode of instability is observed, which is a consequence of the squeezing action of the non-uniform ambient fluid pressure. This is predicted by a linear stability analysis where the dynamic interaction of all the variables is included. We find that strongly elliptic jets will first shift before breaking up into drops. To clearly reveal the underlying physics, we consider the simplified case of a stationary inviscid, incompressible jet.

This study – aimed at shedding light on the role of interfacial curvature on the stability of an elliptic jet – improves our understanding of the physics behind its increased instability and can be extended easily to other non-circular jets. Also, it has been reported that the jet cross section of a magnetic fluid in a transverse external homogenous magnetic field exhibits an elliptic shape (Smirnov and Wolleydt, 1993). Here the imposed magnetic field prevents axis-switching of the elliptic jet. Hence, we expect the physics revealed by our analysis to be equally applicable to such a scenario. The results of this study may also find application in synthesis of ellipsoidal polymer particles. There is a recent interest in making such non-isotropic particles with elliptic shapes. An improved understanding of stability of elliptic jets thus can help optimize the processing conditions for the synthesis of such particles from UV cured oils/polymers.

We first introduce Rayleigh's Work Principle and use it to analyze the static instability of an elliptic jet to two simple subsets of perturbations. This is followed by a linear stability analysis to analyze the dynamic instability of an elliptic jet. We then discuss the results of the two analyses and present our conclusions.

## Rayleigh's Work Principle (RWP)

Rayleigh's Work Principle (RWP) provides a theoretical framework based on work-energy considerations to predict the stability of a given state of fluid flow. Consider a flow configuration whose stability is to be determined. Then, according to RWP, if work is required to perturb this state of flow to a nearby new state, then the original state of flow is deemed to be stable to that perturbation (Johns and Narayanan, 2002). RWP can be viewed as a static stability criterion, since it does not consider the dynamic interactions between variables. Here, we use RWP to predict the cut-off wavenumber associated with capillary instability for an elliptic jet.

To determine whether a cylindrical jet of fluid is stable to a certain perturbation, the work done on the jet in taking it from its base state to the perturbed state is determined. For a static inviscid jet, this work is proportional to the change in its surface area. An increase in surface area corresponds to an increase in surface energy and hence work is required to be done on the jet to take it to the new perturbed state. Conversely, a decrease in surface area of the perturbed state requires work to be done by the jet, which would decrease its energy. A jet according to RWP is therefore stable to all perturbations that increase its surface area and unstable to the ones that decrease it.

We extend Rayleigh's classical analysis of a circular jet and determine the stability of a static elliptic jet. Periodic perturbations of different wavenumbers are imposed along the axis of the jet subject to the condition that the mass (volume) of the jet is conserved. Capillary instability is typically a long – wavelength instability, i.e., long wavelengths (small wavenumbers) are unstable whereas short wavelengths (large wavenumbers) are stable. Therefore, there exists a cut-off wavenumber which corresponds to the transition from unstable to stable wavenumbers. It can be expected that the disturbances with long wavelengths result in a decrease in surface area and are unstable while those with short wavelengths result in an increase in surface area of the jet and are stable. The cut-off wavenumber therefore corresponds to that for which there is no change in surface area.

For the case of a circular cylinder, Rayleigh had examined axisymmetric perturbations to the jet and found the cut-off wavenumber to be $k_c^R = \frac{1}{a}$, where $a$ is the radius of the unperturbed circular jet. For the case of an elliptic jet, we specifically look at two different forms of axially periodic perturbation (a) one that keeps the aspect ratio of the perturbed jet same as the base state. Here the jet dilates and constricts along the axis preserving the aspect ratio (b) one that keeps the focal length of the perturbed jet same as that of the base state, i.e., the jet dilates and constricts preserving the focal length at all axial positions. These two cases correspond to a subset of perturbations that can be imposed on an elliptic jet. We derive an analytical closed form solution for the cut-off wavenumber for each of these cases. Since RWP is based on static energy comparison of a given base state with its perturbed state, it provides a conservative estimate of the cut-off wavenumber compared to that predicted by the dynamic linear stability analysis. In other words, the RWP predicts stability for some wavenumbers which are actually unstable.

### *RWP for an elliptic jet with constant aspect ratio*

In this section, we use RWP to predict the cut-off wavenumber associated with capillary instability for an elliptic jet to perturbations that preserve its aspect ratio. A circular cylindrical coordinate system with $r$, $\theta$ and $z$ being the radial, azimuthal and axial coordinates respectively is used. In this coordinate system, the interface of an elliptic jet is described by