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Two- and three-phase horizontal slug flow simulations using an interface-capturing compositional approach

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ABSTRACT

Progress on the development of a general framework for the simulation of turbulent, compressible, multiphase, multi-material flows is described. It is based on interface-capturing and a compositional approach in which each component represents a different phase/fluid. It uses fully-unstructured meshes so that the latest mesh adaptivity methods can be exploited. A control volume-finite element mixed formulation is used to discretise the equations spatially. This employs finite-element pairs in which the velocity has a linear discontinuous variation and the pressure has a quadratic continuous variation. Interface-capturing is performed using a novel high-order accurate compressive advection method. Two-level time stepping is used for efficient time-integration, and a Petrov–Galerkin approach is used as an implicit large-eddy simulation model. Predictions of the numerical method are compared against experimental results for a five-material collapsing water column test case. Results from numerical simulations of two- and three-phase horizontal slug flows using this method are also reported and directions for future work are also outlined.

Introduction

It is well-known that the flow of water, oil and gas is of considerable practical importance for the oil and gas industry, occurring frequently during oil extraction, transportation, and flow-assurance applications. The physics of such flows involves the strong coupling of a number of mechanisms that include the interaction between the various phases, turbulence, viscosity- and density-contrast-driven instabilities, viscous and gravitational forces. This physics gives rise to complex dynamics that manifests itself through the formation of a wide range of phenomena such as interfacial waves, bubble and droplet creation, re-deposition, and entrapment, which has a direct bearing on the development of different flow regimes.

Bearing in mind that even two-phase gas–liquid flows exhibit complex dynamics, it is immediately apparent that the addition of a third phase will substantially add to this complexity. In such flows, physical phenomena additional to those occurring in two-phase flow play a crucial role. A major difference between two- and three-phase flow is due to the fact that in the latter, the presence of two liquids gives rise to a wider variety of flow patterns (Hall, 1997). Yet three-phase flow of two liquids and gas occurs often, especially in the production of hydrocarbons from oil and gas fields when oil, water, and natural gas flow in the transporting pipelines. The prediction of three-phase gas/liquid/liquid flows is therefore of industrial importance. In two- and three-phase flows, a frequently encountered flow regime is slug flow.

Even in two-phase flows (e.g. gas–liquid flows, for instance) in horizontal and inclined pipes, the generation and evolution of slugs in the slug-flow regime remains relatively poorly understood (Fabre and Line, 1992; King et al., 1998; Ujang et al., 2006). In these flows, the entrainment of gas from the large bubble to form small bubbles in the liquid slug is an important flow feature; the latter process is controlled by micro-scale capillary physics. Since the rate of creation of small bubbles is proportional to the pipe diameter cubed, the gas void fractions in bubble and slug flows converge with increasing pipe diameter. This has led to the observation that there is no 'classical' slug flow in large diameter pipes (Omebere-Iyari et al., 2007).

There are models of slug flow of increasing levels of complexity ranging from one assuming no bubbles (de Cachard and Delhaye, 1996) in the liquid slug (very small diameter pipes/very viscous

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liquids), to those with 29 equations (Fernandes et al., 1983). The first gives reasonable results either in the absence of bubbles, or if the gas fraction of gas in the slug is specified. More than often not, however, flows in hydrocarbon production pipelines are three-phase (oil–water–gas) rather than two-phase, and this additional complexity must be accounted for by any numerical method that aims to provide accurate and reliable predictions of these flows.

In this study, a novel method for detailed modelling of the physical processes that arise in complex multi-phase flows such as slug flows is proposed. The method is based on a multi-component approach that embeds information on interfaces into the continuity equations. A control volume-finite element method mixed formulation is used to discretise the spatial derivatives of the governing equations based on the P_1DG-P_2 (element-wise linear velocity, discontinuous between elements and element-wise quadratic pressure, continuous between elements, with C1 continuity everywhere) element pair (Cotter et al., 2009a; Cotter et al., 2009b). A novel interface-capturing scheme based on high-order accurate compressive advection methods is also used. This involves a down-winding scheme formulated using a high-order finite-element method to obtain fluxes on the control volume boundaries. These fluxes are subject to flux-limiting using a normalised variable diagram approach (Leonard, 1991; Darwish, 1993; Darwish and Moukalled, 2003) to obtain bounded and compressive solutions for the interface; this is essentially equivalent to introducing negative diffusion into the advection equation. The approach also uses a novel two-level time stepping method which allows large time steps to be used while maintaining stability and boundedness. Finally, a non-linear Petrov–Galerkin method is used as an implicit large-eddy simulation model.

This approach is able to simulate highly turbulent multi-phase flows of arbitrary number of phases and equations of state (density, pressure, temperature/internal energy relation), that can naturally represent phase change. The advantages of this approach are that the model is designed for compressible multi-fluid flows with an arbitrary number of fluids/materials, and does not rely on a priority list of materials (common for traditional multi-material models), which often produces spurious solutions (Wilson, 2009). In addition, the component advection equations are embedded into both pressure and continuity equations resulting in local mass balance (within each control volume).

Numerical predictions for the five-material collapsing water column test case are validated against experimental data. Results from numerical simulations of two-dimensional, two- and three-phase horizontal slug flows are then presented. A comparison of these results with experimental observations reveals good agreement and provides an indication of the accuracy, reliability, and efficiency of the numerical approach.

The remainder of this paper is organised as follows. A detailed description of the model is given in Section 'Methodology'. The model evaluation for the five-material collapsing water column test case is presented in Section 'Method validation: collapsing water column test case'. Two- and three-phase horizontal slug flow simulation results are presented in Sections 'Numerical simulation of two-phase horizontal slug flow' and 'Numerical simulation of three-phase horizontal slug flow', respectively. Finally, concluding remarks and directions for future work are given in Section 'Conclusions'.

Methodology

In multi-component flows, a number of components exist in one or more phases. In the present work, one phase is assumed, however, this is easily generalised to an arbitrary number of phases or fluids. For each fluid component i, the conservation of mass is defined as:

$$\frac{\partial}{\partial t}(x_i\rho_i) + \nabla\cdot(x_i\rho_i\mathbf{u}) - Q_i = 0, \quad i = 1,2,\ldots,\mathcal{N}_c, \tag{1}$$

where $t, \mathbf{u}$ and Q_i are the time, velocity vector and mass source term, respectively, and ρ_i is the density of component i. In Eq. (1), x_i is the mass fraction of component i, where $i = 1,2,\ldots,\mathcal{N}_c$, and $\mathcal{N}_c$ denotes the number of components, which is subject to the following constraint:

$$\sum_{i=1}^{\mathcal{N}_c} x_i = 1. \tag{2}$$

The equations of motion of a compressible viscous fluid may be written as:

$$\frac{\partial}{\partial t}(\rho\mathbf{u}) + \nabla\cdot(\rho\mathbf{u}\mathbf{u}) = \nabla\cdot\overline{\overline{\sigma}} - \nabla p + \rho g\mathbf{k}, \tag{3}$$

where $\overline{\overline{\sigma}}$ is the deviatoric stress tensor, p is the pressure, the bulk density is $\rho = \sum_i^{\mathcal{N}_c} x_i\rho_i, g$ is the gravitational acceleration, and $\mathbf{k}$ is a unit vector pointing in the direction of gravity. Assuming $Q_i = 0$, testing with control volume basis functions M_m, and applying integration by parts and using the θ-time stepping method for the advection terms, Eq. (1) can be expressed as:

$$\int_{V_m} M_m\left(\frac{x_{im}^{n+1}\rho_{im}^{n+1} - x_{im}^n\rho_{im}^n}{\Delta t}\right)dV + \int_{\Gamma_m}\left[\theta_i^{n+\frac{1}{2}}x_i^{n+1}\rho_i^{n+1}\mathbf{n}\cdot\mathbf{u}^{n+1} + \left(1-\theta_i^{n+\frac{1}{2}}\right)x_i^n\rho_i^n\mathbf{n}\cdot\mathbf{u}^n\right]d\Gamma = 0, \quad m = 1,2,\ldots,\mathcal{M}, \tag{4}$$

where $\theta(\in\{0,1\})$ is the implicitness parameter, $\mathbf{n}$ is the outward-pointing unit normal vector to the surface of the control volume m, $\mathcal{M}$ is the number of control volumes, n represents the time level; here, V_m and Γ_m represent the volume and surface area of the control volume m, respectively. Dividing Eq. (4) by ρ_{im}^{n+1} and then summing over all components leads to the global mass conservation:

$$\int_{\Gamma_m}\left[\Theta_m^{n+\frac{1}{2}}\mathbf{n}\cdot\mathbf{u}^{n+1} + (1-\Theta)_m^{n+\frac{1}{2}}\mathbf{n}\cdot\mathbf{u}^n\right]d\Gamma = \int_{V_m} M_m\mathcal{S}_m^{n+\frac{1}{2}}dV, \quad m = 1,2,\ldots,\mathcal{M}. \tag{5}$$

Eq. (5) is also bounded by the component-mass constraints:

$$\sum_{i=1}^{\mathcal{N}_c} x_{im}^n = \sum_{i=1}^{\mathcal{N}_c} x_{im}^{n+1} = 1, \tag{6}$$

and the absorption term $\mathcal{S}_m^{n+\frac{1}{2}}$ is:

$$\mathcal{S}_m^{n+\frac{1}{2}} = -w_c\frac{1}{\Delta t}\left(1 - \sum_{i=1}^{\mathcal{N}_c} x_{im}^{n+1}\right) - \sum_{i=1}^{\mathcal{N}_c}\left(\frac{x_{im}{}^{n+1}\rho_{im}^{n+1} - x_{im}^n\rho_{im}^n}{\rho_{im}^{n+1}\Delta t}\right). \tag{7}$$

The term $w_c(\in\{0,1\})$ helps enforcing the component-mass constraint Eq. (6): $w_c = 1$ represents a full correction but may lead to an unstable scheme, whereas $w_c = 0$ represents a no-correction operation. Here, a compromise $w_c = 0.5$ is used. So, the first term in the *rhs* of Eq. (7) applies the summation constraint Eq. (6, as $w_c \to 1$) and effectively replaces the summation of the time implicit term on the *rhs* of Eq. (4) with $1/\Delta t$. By ensuring positivity of the effective absorption in the mass conservation associated with the w_c term, the scheme's stability may be improved. This means that the first term on the *rhs* of Eq. (7) is replaced with:

$$-w_c\frac{1}{\Delta t}\max\left\{1 - \sum_{i=1}^{\mathcal{N}_c} x_{im}^{n+1}, 0\right\}.$$

The space/time flux-limiting functions (from Eq. (5)) are defined as: