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Control volume finite element modelling of segregation of sand and granular flows in fluidized beds

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ABSTRACT

This paper presents a computational methodology for the two and three dimensional numerical simulation of dense and dilute dispersed particles in multiphase gas–solid and liquid–solid flows. The model equations are based on the two fluids approximation, with closure terms for the fluid–solid drag interaction forces and the additional dense collisional terms arising from particle–particle interactions under the kinetic theory. These equations are discretized and solved using a novel unstructured mesh control volume-finite element framework, with an anisotropic mesh adaptivity capability. The methodology is applied to study the transport of sand particles of various sizes in fluidized beds, as well as particle segregation in a polydisperse system containing three solid particle sizes.

Introduction

The need to understand and predict the behaviour of dispersed solid particles within a continuum phase has a long history, and appears in a number of industrial applications, from the pneumatic transport of coal in power plants (Geldart and Ling, 1990), to the application of catalysts in chemical reactors (Douek et al., 1995), to sand deposition (Doron et al., 1997) and abrasion in oil and gas pipelines. Direct numerical simulation of such systems is difficult when the number of dispersed particles may be in the millions, and when particles can differ in size and shape.

Numerical models for such systems may be divided into categories depending on the frame of representation applied to the continuum fluid and to the particles themselves. These can range from the fully-Lagrangian methods of techniques such as smoothed particle hydrodynamics (Xiong et al., 2011), to Eulerian–Lagrangian methods such as the discrete element method in monodispersed and polydispersed systems (Sakai et al., 2010; Sakai et al., 2012; Sakai et al., 2014; Dosta et al., 2012; Feng and Yu, 2010), to the Eulerian–Eulerian approach applied in this paper. Here we follow the philosophy of the two-fluid model (Enwald et al., 1996), effectively treating the lengthscale-filtered solid concentration as a second, miscible, fluid phase (Hewitt and Vassilicos, 2005). Closures are applied for the fluid–solid drag forces and for the collision energy exchange occurring within solid–solid particle interactions, with the form of the latter usually, as here, derived from an application of a theory analogous to the kinetic theory of gases (Chapman and Cowling, 1970).

Such methods have previously been applied with great success to modelling of flows in risers (Neri and Gidaspow, 2000) and fluidized beds (Taghipour et al., 2005). However, it has been noted that the ability to capture correctly the behaviour of transient phenomena is strongly dependent on the numerical resolution being adequate in the vicinity of the feature of interest (Wang et al., 2010). This suggests the feasibility of applying mesh adaptivity to generate accurate solutions to such systems efficiently.

Mesh adaptivity and the related field of adaptive mesh refinement are increasingly popular methods in computational fluid dynamics. Methods range from a global remeshing of the entire domain (Peraire and Morgan, 1997), to various local or nested refinement algorithms, modifying the mesh structure only in the

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regions of interest. Criteria for the definition of a good mesh can range from *a priori* or *a posteriori* error estimates (Verfürth, 1994) to phenomenological constraints based on the expected appearance of important phenomena (Weller, 2008).

The structure of this paper is as follows: in the following section we describe the granular flow equations and closures being used to model the multiphase physics, and the control volume finite element method being applied to discretize them. We then go on to describe the algorithm used to control mesh adaptivity. In the next section we present the results of an application of the model and introduce test cases for flows in monodispersed fluidized beds (on both fixed and adaptive meshes) and polydispersed fluidized beds (on fixed meshes only). The mono dispersed simulations are compared with experimental data. Finally we draw a brief conclusion.

Model governing equations and discretization

We begin by listing the governing equations applied to simulate the fluidized bed systems. Although similar equation sets have been used previously by the authors to simulate monodisperse fluidized bed systems (Pain et al., 2002; Gomes et al., 2007) we here present additional terms which allow for multiple solid particle size distribution classes. The flow is modelled using an Eulerian–Eulerian incompressible multiphase approach. Separate continuity and momentum balance equations are solved for each phase,

$$\frac{\partial}{\partial t}\left(\rho_g\alpha_g\right)+\nabla\cdot\left(\rho_g\alpha_g\boldsymbol{u}_g\right)=0, \tag{1}$$

$$\frac{\partial}{\partial t}(\rho_i\alpha_i)+\nabla\cdot(\rho_i\alpha_i\boldsymbol{u}_i)=0,\qquad i\in[1,\ldots N], \tag{2}$$

$$\frac{\partial}{\partial t}\left(\rho_g\alpha_g\boldsymbol{u}_g\right)+\nabla\cdot\left(\rho_g\alpha_g\boldsymbol{u}_g\boldsymbol{u}_g\right)=-\alpha_g\nabla p_g+\nabla\cdot\underline{\boldsymbol{\tau}_g}+\boldsymbol{F}_g, \tag{3}$$

$$\frac{\partial}{\partial t}(\rho_i\alpha_i\boldsymbol{u}_i)+\nabla\cdot(\rho_i\alpha_i\boldsymbol{u}_i\boldsymbol{u}_i)=-\alpha_i\nabla p_g+\nabla\cdot\underline{\boldsymbol{\tau}_i}+\boldsymbol{F}_i. \tag{4}$$

Here the index g denotes filtered properties ascribed to the gas phase, while the index i denotes a quantity ascribed to a solid phase (a single phase for monodisperse system modelling, many phases in the polydisperse case). The other variables in the continuity equations are (constant) material densities, ρ, phase volume fraction, α and phase velocity, $\boldsymbol{u}$. The further quantities introduced in the momentum equations are the gas phase pressure, p_g, stress tensor, $\underline{\boldsymbol{\tau}}$ and body forces, $\boldsymbol{F}$. Conservation of total volume in the system implies a further constraint,

$$\alpha_g+\sum_j\alpha_j=1. \tag{5}$$

For the gas phase, the stress tensor takes the usual form,

$$\underline{\boldsymbol{\tau}_g}=\frac{\mu_g}{2}\left(\nabla\boldsymbol{u}_g+(\nabla\boldsymbol{u}_g)^T-\frac{2}{3}\nabla\cdot\boldsymbol{u}_g\right), \tag{6}$$

for a prescribed gas viscosity μ_g. In contrast, the solid stress tensor is more complex, containing closure terms representing the additional forces arising from the interaction (in the dense limit) of the individual solid particles assigned to a particular phase:

$$\underline{\boldsymbol{\tau}_i}=\left(-p_i\underline{\boldsymbol{I}}+\xi_i[\nabla\cdot\boldsymbol{u}_i]\underline{\boldsymbol{I}}+\frac{\mu_i}{2}\left[\nabla\boldsymbol{u}_i+(\nabla\boldsymbol{u}_i)^T-\frac{2}{3}\nabla\cdot\boldsymbol{u}_i\right]\right). \tag{7}$$

Following Gidaspow (1994), the additional solid pressure term, p_i appearing on the diagonal is assumed to take a form arising from an application of the kinetic theory to finite diameter particles. Assuming constant material properties we obtain

$$p_i=\rho_i\alpha_i\Theta\left[1+2(1+e)g_0\sum_{j=1}^{N}\alpha_j\right], \tag{8}$$

where Θ is an additional modelled quantity, commonly termed the granular temperature, parameterizing the distribution of the energies of the individual solid particles around their local spatially filtered value. Although in general the granular temperature of different phases may differ and must be formally related (see for example van Sint Annaland et al., 2009, for such a model), we neglect such terms here under the assumption that the particle concentrations remain dense enough that variations between phases equilibrate fast. The parameter e is a restitution coefficient, describing the energy loss in particle–particle collisions and the function g_0, termed the radial distribution function, parameterizes the likelihood of particle–particle interaction at a given concentration,

$$g_0(\alpha)=\frac{3}{5}\left(1-\left(\frac{\sum_{i=1}^{N}\alpha_i}{\alpha_{\max}}\right)^{\frac{1}{3}}\right)^{-1}. \tag{9}$$

Here $\alpha_{\max}$ is a parameter limiting the maximum allowable particle packing fraction. Note that in this formulation we make no correction in the polydisperse limit, although it will be recognized that the maximum packing fraction of a binary mixture of two disparate particle sizes will be in excess of this single particle limit (Zhu et al., 2008).

The solid viscous compressional and shear viscosities are similarly parameterized as

$$\xi_i=\frac{4}{3}\rho_i\alpha_i d_i\left(g_0\sum_j\alpha_j\right)(1+e)\sqrt{\frac{\Theta}{\pi}}, \tag{10}$$

$$\mu_i=\frac{4}{5}\rho_i\alpha_i d_i\left(g_0\sum_j\alpha_j\right)(1+e)\sqrt{\frac{\Theta}{\pi}}, \tag{11}$$

where d_i is the typical particle diameter for the ith phase i,e the particles are effectively assumed to be perfectly spherical. A summation is applied here under the assumption that the pseudo-viscous energy lost through a collision is not a function of the particle classes colliding. Finally, the body force terms are

$$\boldsymbol{F}_g=-\sum_{j=1}^{N}\beta_j^{(g)}(\boldsymbol{u}_g-\boldsymbol{u}_j)-\rho_g\alpha_g\boldsymbol{g}, \tag{12}$$

$$\boldsymbol{F}_i=\beta_i^{(g)}(\boldsymbol{u}_g-\boldsymbol{u}_i)+\sum_{j=1}^{N}\beta_{ij}(\boldsymbol{u}_j-\boldsymbol{u}_i)-\rho_i\alpha_i\boldsymbol{g}, \tag{13}$$

where we choose the popular Wen and Yu (1966) model for gas-particle drag,

$$\beta_i^{(g)}=\frac{3}{4}C_D\frac{\alpha_i\rho_g|\boldsymbol{u}_g-\boldsymbol{u}_i|}{d_i}\alpha_g^{-2.65}, \tag{14}$$

where the coefficient is prescribed as

$$C_D=\begin{cases}\frac{24}{\mathrm{Re}_i}\left(1+0.15\mathrm{Re}_i^{0.687}\right) & \mathrm{Re}_i<1000,\\ \frac{24}{1000}\left(1+0.15[1000]^{0.687}\right)\approx 0.44 & \mathrm{Re}_i\geqslant 1000,\end{cases} \tag{15}$$

using a particle Reynolds number defined as

$$\mathrm{Re}_i=\frac{\alpha_g\rho_g|\boldsymbol{u}_g-\boldsymbol{u}_i|d_i}{\mu_g}. \tag{16}$$

Experimentally, the Wen and Yu model has proven a good phenomenological fit for the drag exerted in the monodisperse limit for dilute models (Delebarre, 2004). The more recent development of closures based on ensembles of high resolution lattice Boltzmann simulations (Benyahia et al., 2006; Yin and Sundaresan, 2009; Tenneti et al., 2011) offer possibilities for investigation in the future, particularly for polydispersed fluid–solid (Cello et al., 2010) and fluid–fluid flows.