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Prediction of a particle-laden turbulent channel flow: Examination of two classes of stochastic dispersion models

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ABSTRACT

Nowadays, two families of stochastic models are mainly used to predict the dispersion of inertial particles in inhomogeneous turbulent flows. This first one is named "normalized model" and the second one "Generalized Langevin Model (GLM)". Nevertheless, the main differences between the normalized and GLM models have not been thoroughly investigated. Is there a model which is more suitable to predict the particle dispersion in inhomogeneous turbulence? We propose in the present study to clarify this point by computing a particle-laden turbulent channel flow using a GLM-type model, and also a normalized-type model. Particle statistics (such as concentration, mean and rms particle velocity, fluid-particle velocity covariances) will be provided and compared to Direct Numerical Simulation (DNS) data in order to assess the performance of both dispersion models. It will be shown that the normalized dispersion model studied can predict correctly the effect of particle inertia on some dispersion statistics, but not on all. For instance, it was found that the prediction of the particle kinetic shear stress and some components of the fluid-particle covariance is not physically acceptable.

1. Introduction

In order to describe the transport of inertial particles in a turbulent flow, the point-force approximation is usually used. The particle equation of motion can be then solved if the instantaneous fluid velocity at particle location (fluid velocity seen by particles) is known. Several methods can be used to properly determine this velocity. The first possibility is to use Direct Numerical Simulation (DNS). It gives the most accurate estimation of the fluid velocity seen by particles. Nevertheless, it necessitates high computational resources. When the computational cost of this latter technique is too high, macroscopic numerical simulation such as Reynolds-Averaged Navier–Stokes (RANS) can be considered. To describe the motion of solid particles in a turbulent flow using a RANS method, the random nature of the velocity along inertial particle trajectories has to be reconstructed, for instance, using Langevin-type models since only mean quantities (such as the mean velocity and some of the mean turbulent characteristics) of the carrier phase are given by the RANS method.

According to the literature, two families of Langevin-type models emerge. The first one is generally named "normalized models". They are derived from the pioneer work of Wilson et al. (1981) in which a model that avoids the non-physical accumulation of tracer particles in inhomogeneous turbulent flows (spurious drift effect) is proposed. In order to get rid of the spurious drift effect, an ad hoc term was added to the Langevin equation. The consistency of their stochastic model with the Navier–Stokes equations was not considered while it was the starting point of the derivation of the Generalized Langevin Model (GLM) proposed by Pope (1983).

The normalized and GLM models, developed for predicting the trajectory of fluid particles, were later extended to determine the fluid velocity seen by inertial particles by Sawford and Guest (1991) and Simonin et al. (1993), respectively. Nowadays, these two families of models are still used to model the motion of inertial particles in inhomogeneous turbulent flows. Nevertheless, the main differences between the normalized and GLM models have not been thoroughly investigated. Is there a model which is more suitable to predict the particle dispersion in inhomogeneous turbulence from an engineering point of view ?

We propose in the present study to clarify this point by computing a particle-laden turbulent channel flow using a GLM-type model (Arcen and Tanière, 2009) and a specific form of a normalized-type model (Dehbi, 2010), the parameters of these models being determined from DNS data. Particle statistics (such as

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concentration, mean and rms particle velocity, and fluid-particle velocity covariances) are provided and compared to DNS data in order to assess the performance of both models.

The present paper is organized as follows: the stochastic models are briefly described; the configuration and numerical simulation of the gas–solid turbulent flow are presented in the next section; then we analyze the results given by the normalized stochastic model through a comparison with data previously obtained by DNS and with a GLM stochastic model (Arcen and Tanière, 2009). Finally, the last section is devoted to concluding remarks.

2. Brief description of the dispersion stochastic models

From a literature review about stochastic dispersion models for non-homogeneous turbulent flows, it can be noticed that the existing models are extension of models previously derived to predict the motion of fluid particles in a turbulent flow. It should also be noted that most of these stochastic models for the fluid particle velocity were originally developed for homogeneous isotropic turbulence and then extended to inhomogeneous turbulent flows. Considering homogeneous isotropic turbulence, the time increment of the instantaneous fluid velocity, du_i, can be described by the following stochastic process:

$$du_i = -\frac{u_i}{T_L}dt + \left(\frac{2\left\langle u_i'^2\right\rangle}{T_L}\right)^{\frac{1}{2}} dW_i, \quad (1)$$

where the input parameters for the above Langevin model are the variance of the velocity, $\langle u_i'^2\rangle$($\langle\cdot\rangle$ denotes averaged quantities), and the Lagrangian integral timescale, noted T_L. $W_i(t)$ is a set of independent Wiener processes which have increments normally distributed with zero mean and variance dt.

Since the 1980s, it has been shown that the direct use of Eq. (1) to predict the diffusion of fluid particles in non-homogenous turbulent flows can lead to non-physical effects. Such a model does not give realistic results since particle concentration tends to increase in regions with low velocity variance (spurious drift effect). The works conducted by Wilson et al. (1981); Legg and Raupach (1982) or by Pope (1983) give some solution that avoids the non-physical fluid particle accumulation. A corrective term can be directly introduced in the stochastic equation in order to avoid the spurious drift effect. This is the way chosen by Wilson et al. (1981) and Legg and Raupach (1982). For instance, reduced to the wall-normal direction (x_2) in a boundary layer (where the mean wall-normal velocity is supposed to be equal to zero), the normalized formulation proposed by Wilson et al. (1981) is:

$$d\left(\frac{u_2}{\sigma_2}\right) = -\frac{u_2}{\sigma_2}\frac{dt}{T_L} + \left(\frac{2}{T_L}\right)^{\frac{1}{2}} dW + \frac{d\sigma_2}{dx_2}dt, \quad (2)$$

where $\sigma_2^2 = \left\langle u_2'^2\right\rangle$. The last term of this equation is the correction term which avoids the spurious drift effect. It corresponds to a mean force due to the action of the mean pressure gradient on fluid particles.

Following a different path, Pope (1983) derived a functional form of the Langevin-type model for the instantaneous fluid velocity increment which is consistent with the averaged Navier–Stokes equations. It takes the following form:

$$du_i = \left[-\frac{1}{\rho_f}\frac{\partial\langle p\rangle}{\partial x_i} + \nu\nabla^2\langle U_i\rangle\right]dt + G_{ij}[u_j - \langle u_j\rangle]dt + B_{ij}dW_j, \quad (3)$$

where G_{ij} has an inverse time dimension, ρ_f is the fluid density, ν is the fluid kinematic viscosity. It should be noted that the pressure gradient appears naturally and correctly in opposite to the previous normalized model (Eq. (2)). This group of models is named Generalized Langevin Models, noted GLM, where the drift and diffusion parameters, G_{ij} and B_{ij} respectively, have to be specified. The increment of the fluctuating fluid velocity ($du_i' = du_i - d\langle u_i\rangle$) can be also derived:

$$du_i' = \left[\frac{\partial\langle u_i'u_j'\rangle}{\partial x_j} - u_j'\frac{\partial\langle u_i\rangle}{\partial x_j}\right]dt + G_{ij}u_j'dt + B_{ij}dW_j. \quad (4)$$

Pope (1987) provides an important algebraic relation between the drift and diffusion terms which ensures the Eulerian consistency with the Reynolds stress transport equation. From Eq. (4), it can be shown that the GLM is consistent to second order and for any turbulent flows, if the drift and diffusion parameters satisfy the relation:

$$G_{jk}\langle u_i'u_k'\rangle + G_{ik}\langle u_j'u_k'\rangle + B_{ik}B_{jk} = +\nu\frac{\partial^2}{\partial x_k\partial x_k}\langle u_i'u_j'\rangle - \frac{1}{\rho_f}\left[\left\langle u_i'\frac{\partial p'}{\partial x_j}\right\rangle + \left\langle u_j'\frac{\partial p'}{\partial x_i}\right\rangle\right] - 2\nu\left\langle\frac{\partial u_i'}{\partial x_k}\frac{\partial u_j'}{\partial x_k}\right\rangle. \quad (5)$$

Based on these ideas about the modeling of fluid particle velocity, stochastic models were later developed to predict the fluid velocity at solid particle location in two-phase turbulent flow modeling. To determine the trajectory of a solid particle using the point force approximation, the particle equations of motion have to be solved. Considering that the drag force is only of importance, these equations take the following form:

$$\begin{cases} \dfrac{dx_{p,i}}{dt} = v_{p,i}, \\ \dfrac{dv_{p,i}}{dt} = \dfrac{(\tilde{u}_i - v_{p,i})}{\tau_p}, \end{cases} \quad (6)$$

where $x_{p,i}$ and $v_{p,i}$ are the particle position and velocity, τ_p is the particle relaxation time which is expressed in terms of the drag coefficient and of the magnitude of the relative velocity, and $\tilde{u}_i = u_i(\mathbf{x}_p, t)$ is the fluid velocity at the particle location. Normalized or GLM models were extended to model this velocity (i.e. $\tilde{u}_i$). We focus here specially on the GLM-type model proposed by Arcen and Tanière (2009) which is known to provide an accurate prediction of particle dispersion in a non-homogeneous turbulent flow. It should be noted that this GLM-type model is also compatible with the transport equation of the drift velocity (mean fluctuating fluid velocity at particle location), noted $\langle\tilde{u}_i'\rangle = \langle\tilde{u}_i - \langle u_i\rangle\rangle$, in the limits of low and high particle inertia. The form of this model is:

$$d\tilde{u}_i = \left[-\frac{1}{\rho_f}\frac{\partial\langle p\rangle}{\partial x_i} + \nu\frac{\partial^2\langle u_i\rangle}{\partial x_j\partial x_j} + (v_{p,j} - \tilde{u}_j)\frac{\partial\langle u_i\rangle}{\partial x_j} + G^*_{ij}(\tilde{u}_j - \langle u_j\rangle) + \frac{\partial}{\partial x_k}\left(\langle\tilde{u}_i'v_{p,k}'\rangle - \langle u_i'u_k'\rangle\right)\right]dt + B^*_{ij}dW_j. \quad (7)$$

Its fluctuating counterpart is given by:

$$d\tilde{u}_i' = \left[G^*_{ij} - \partial\langle u_i\rangle/\partial x_j\right]\tilde{u}_j'dt + B^*_{ij}dW_j + \frac{\partial\left\langle\tilde{u}_i'v_{p,j}'\right\rangle}{\partial x_j}dt. \quad (8)$$

The drift and diffusion parameters, G^*_{ij} and B^*_{ij}, are different from G_{ij} and B_{ij}. Nevertheless they have to become identical for $\tau_p \to 0$. The values of the drift and diffusion parameters are linked via Eq. (5).

Concerning the normalized model, we examined the model recently used by Dehbi (2010) to predict the motion of particles in a turbulent channel flow. This stochastic model was derived from the work of Hanratty and its co-workers (e.g. Iliopoulos et al., 2003), and from the proposal of a drift correction term which takes into account the particle inertia (Bocksell and Loth, 2001; Bocksell and Loth, 2006). This model, written for the fluid fluctuating velocity at solid particle location, is:

$$d\left(\frac{\tilde{u}_i'}{\sigma_i}\right) = -\left(\frac{\tilde{u}_i'}{\sigma_i}\right)\frac{dt}{\tau_i} + \sqrt{\frac{2}{\tau_i}}dW_i + \frac{\partial}{\partial x_2}\left(\frac{\langle u_i'u_2'\rangle}{\sigma_i}\right)\frac{dt}{1+St}, \quad (9)$$