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Two-dimensional numerical investigation on the dynamics of ligament formation by Faraday instability

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A B S T R A C T

Ligament formation from the surface of a horizontal liquid layer subject to a vertical vibration (Faraday instability) is a crucial part of the atomization process because it is the transition phase for droplet generation. Based on the numerical solutions of the two-dimensional incompressible Euler equations for a prototype Faraday instability flow, we explored physically how a liquid ligament that is dynamically free from the vibrating liquid layer and behaves like a jet can be produced. According to linear theory, the suction of liquid from the trough portion to the crest portion creates an amplified crest. The amplified crest is always pulled back to the liquid layer in linear theory, no matter how largely the surface deforms; thus, a dynamically freed ligament never forms. However, under nonlinear conditions produced by large surface deformation, the impinging liquid flow from the trough portion enhances the pressure at the high crest (ligament) root. This pressure enhancement has two major effects. First, it reduces the amount of liquid sucked from the trough portion, which abates the increase in the crest height compared with that associated with linear theory. Second, it forms a local maximum pressure at the crest root; in this case, the ligament above this location becomes dynamically free from the motion of the bottom substrate in the laboratory reference frame. Liquid elements continuously enter the dynamically freed liquid region, producing a slender ligament from the liquid layer.

1. Introduction

Liquid atomization plays an important role in a wide range of industrial applications. For example, in an automotive engine, especially a diesel engine, the combustion efficiency and emission quality are highly dependent on the fuel spray and atomization characteristics. Other fields where liquid atomization is applied include pharmaceutical emulsification, encapsulation, and ink-jet printing. The increasing importance of liquid atomization in various industrial processes requires a better understanding of the underlying physical dynamics to further enhance the process.

Many techniques have been used to realize liquid atomization. One practical method is subjecting a body of liquid to a vertical vibration to produce droplets from its surface. When the forcing acceleration amplitude is small, standing waves are often observed on the surface. This phenomenon was first studied by Faraday (1831); thus, it is referred to as a "Faraday instability" and the resultant surface standing waves are called "Faraday waves". A theoretical analysis of a Faraday instability in the linear regime was carried out by Benjamin and Ursell (1954) for ideal fluids. They obtained an instability chart based on two parameters X and Y (defined by Eq. (2)), which are related to the forcing displacement amplitude and the frequency. Faraday waves and associated nonlinear effects were reviewed by Miles and Henderson (1990). Kumar (1996) and Kumar and Tuckerman (1994) studied the effects of viscosity on atomization using linear stability analysis. When the forcing acceleration amplitude is enhanced sufficiently beyond Faraday wave formation, ligaments (or spikes) disintegrating into droplets are created from the liquid surface. One typical application of such a technique is the so-called "ultrasonic atomization", the forcing frequency of which is on the order of 10 kHz to 1 MHz. This phenomenon was first investigated by Wood and Loomis (1927). Experimental research was then conducted by Lang (1962), who derived a correlation between the median droplet diameter and the forcing frequency in the frequency range of 10–800 kHz. Rajan and Pandit (2001) studied the effects of the liquid properties including the flow rate, viscosity, density, and surface tension, and the ultrasonic properties including the forcing displacement amplitude and frequency to predict the droplet size. Al-Sueimani et al. (1999), Yule and Al-Suleimani (2000), Al-Suleimani and Yule (2002)) investigated the disorder in surface standing waves using high-speed imaging techniques to explain the random ejection locations and the range of droplet sizes generated by ultrasonic atomization. Goodridge et al. (1996, 1997, 1999) studied the threshold conditions for droplet ejection

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by a Faraday instability at low frequencies below 100 Hz. The threshold forcing acceleration amplitude is dependent on the surface tension and forcing frequency for low-viscosity liquids, and on the viscosity and forcing frequency for high-viscosity liquids.

Besides atomizing the liquid layer by vertical vibration (Faraday instability), similar atomization techniques that exploit the vibration have been proposed recently. Tsai et al. (1996, 1997) invented a new spray technique known as "ultrasound-modulated two-fluid (UMTF) atomization", which utilizes the resonance between the liquid capillary waves generated by ultrasound and those generated by air to create a relatively narrow droplet-size distribution. James et al. (2003a,b) and Vukasinovic et al. (2004, 2007) introduced a method known as "vibration-induced drop-atomization (VIDA)" to atomize a parent liquid drop resting on a vibrating horizontal diaphragm into a spray. Moreover, Qi et al. (2008) and Tan et al. (2010) numerically and experimentally studied surface acoustic wave (SAW) atomization, which rapidly generates micron-size aerosol droplets.

The nonlinear nature of the atomization phenomenon introduces great complexity into its mathematical treatment. Moreover, due to its inherently small temporal and spatial scales, it is difficult to determine the underlying atomization mechanism solely by experiments. Recent improvements to computer capacity and numerical schemes allow information to be extracted from detailed flow-field data, which are difficult to measure experimentally. As a result, increasingly more numerical investigations have been conducted to study the fundamental physics of atomization (Al-Sueimani et al., 1999; Wright et al., 2000; Al-Suleimani and Yule, 2002; James et al., 2003b; Shinjo and Umemura, 2010; Takagi and Matsumoto, 2011).

Any atomization process can, in general, be divided into two sub-processes: ligament (or spike) formation from the liquid surface, and disintegration of the ligament into droplets. The mechanism of the latter process, studied extensively through liquid column disintegration research (Eggers, 1997; Eggers and Villermaux, 2008; Umemura, 2011), does not crucially depend on the way the ligament is formed. The disintegration of the ligament into droplets is not the focus of this study.

For a liquid jet emitted from a nozzle, ligaments can be formed by the action of gaseous suction pressure/shear stress on the crest of the liquid jet surface (Shinjo and Umemura, 2010). In a Faraday instability, the gaseous effect is not essential; this suggests that some other mechanism must be required for ligament formation.

The major forces determining the dynamics of large surface deformation (ligaments or spikes at the liquid surface) or breakups induced by vibration are the inertial force, viscous force, gravitational force, and capillary force. Yule and Al-Suleimani (2000) experimentally studied the disorderliness of droplet formation. They claimed that the Froude number (inertial force/gravitational force) does not affect the droplet size or droplet formation process. James et al. (2003b) numerically studied the effect of the Reynolds number (inertial force/viscous force), the dimensionless forcing acceleration amplitude (inertial force/capillary force), and the Bond number (gravitational force/capillary force) on the volume and velocity of the ejected drop and the time of ejection in VIDA at low forcing frequencies. Vukasinovic et al. (2004) experimentally studied the mechanism of free surface breakup in VIDA at forcing frequencies on the order of 1 kHz. They established the dependence of the breakup time and the unbroken spike length on the capillary number (viscous force/capillary force). Donnelly et al. (2004) experimentally studied the droplet diameter distribution at frequencies on the order of 1 MHz. They determined that the relationship between the droplet size and forcing frequency follows an inviscid scaling law. Qi et al. (2008) experimentally and numerically studied SAWs; they determined that the growth of the interfacial wave was caused by a destabilizing effect due to the large acoustic irradiation overwhelming the stabilizing and restoring effect of the capillary force.

However, these previous studies have focused mainly on the dynamics or mechanisms of droplet formation. Studies of the detailed dynamics of ligament formation from a liquid layer by a Faraday instability, especially numerical attempts, have been limited. Thus far, it has been established that the inertial effect plays an essential role in ligament formation. To quantify the condition for spray formation by a Faraday instability, we must reveal the physical mechanism of how the inertial effects produce large surface deformation that is dynamically free from the motion of the bottom substrate by fully analyzing the detailed velocity and pressure fields. The term "ligament" in this study is thus specifically defined as a large liquid surface deformation that is dynamically free from the vibrating bottom substrate and results in disintegrated droplets having outward velocities (i.e., the condition for spray formation). To our knowledge, there is no literature addressing this mechanism in detail. Therefore, we conducted the present numerical research.

The rest of this paper is organized as follows. The physical model and the numerical setup are described in Section 2. Test calculations are conducted to validate the numerical code in both the linear and nonlinear regimes in Section 3. The dynamics of the ligament formation are analyzed in detail based on a prototype in Section 4. Finally, the results of this study are summarized in Section 5.

2. Problem specification and numerical method

2.1. Physical model used for calculation

We consider a liquid layer resting horizontally on a substrate, subject to a vertical vibration of standard sinusoidal displacement, $\Delta_0 \sin(\Omega t)$, where Δ_0 is the amplitude of the forcing displacement, Ω is the forcing frequency, and t is the time. Before proposing a physical model for the calculation, we concisely review the inviscid linear theory derived by Benjamin and Ursell (1954) to introduce the system parameters described in the introduction.

Any surface deformation can be expressed by a Fourier series in the horizontal coordinate x. Each Fourier component $a = \delta(t)\sin(kx)$ (where k is the wavenumber) is independent of the others in the linear regime. The amplitude $\delta(t)$ obeys the Mathieu equation

$$\frac{d^2\delta}{d\hat{t}^2} = (X \sin \hat{t} - Y) \cdot \delta, \tag{1}$$

where $\hat{t} = \Omega t$. The linear stability of the liquid layer is determined by two parameters X and Y, defined as

$$X = k\Delta_0 \tanh(ky_0) \quad \text{and} \quad Y = \left(\frac{\omega}{\Omega}\right)^2 = \frac{\sigma k^3 \tanh(ky_0)}{\rho_l \Omega^2}, \tag{2}$$

where y_0 is the thickness of the liquid layer, σ is the surface tension coefficient, and ρ_l is the liquid density. For each parameter pair (X, Y), we obtain a solution of the surface deformation amplitude $\delta(t)$, from the behavior of which we can determine whether or not the motion of the free surface is stable and the response of the surface oscillation is sub-harmonic, harmonic, or higher-order harmonic. The instability conditions derived from linear theory are charted in Fig. 1 as hatched regions.

An experiment with certain values of the forcing displacement amplitude and frequency (Δ_0, Ω) can be described by a cubic curve, as shown in Fig. 1, by eliminating k from Eq. (2). Under the assumption $\tanh(ky_0) \to 1$, the curve is expressed as

$$Y = X^3 \left(\frac{\sigma}{\rho_l \Delta_0^3 \Omega^2}\right) = \frac{X^3}{\beta}, \tag{3}$$