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Construction of conductive pathways using Genetic Algorithms and Constructal Theory

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ABSTRACT

In present study, two methodologies are proposed for the construction of high conductivity pathways inserted in a square plate of low conductivity and with volumetric internal heat generation. Constructal Design and Genetic Algorithms were used to reach main goal of this work that is to minimize global thermal resistance that occurs in the volume, facilitating access to the heat flow through the conductive pathways, which are connected to a heat sink. Total volume of the plate and that occupied by the material of high thermal conductivity are fixed as well as the thermal ratio between the conductivities of the materials. First methodology facilitates the construction of the paths in an evolutionary way using all available material, while second methodology, also using the idea of evolution, gradually distributes (a percentage to each evolution) the material, incrementally. Forms found for the paths have great similarity to the tree-shaped branched forms found in nature. When ratio of high to low conductivity material is small, the high conductive material tends to accumulate closer to the heat sink creating thicker paths, on the other hand, using a larger conductivity ratio, these paths are thinner and tend to move away from the heat sink. Although there is no universal optimal form for this problem, results presented offer an efficient performance, characterized by the constant minimization of temperature as the material is added and/or the Genetic Algorithm (GA) is applied. The use of GA offers great computing advantages. In addition, some of obtained results presented better performance, compared with I-shape case studied in the literature.

1. Introduction

Volume to Point (VP) problem is a fundamental problem applied to electronic cooling which has been largely studied in literature [1–8]. With the increase of power and decrease in size of electronic devices, cooling problem has become of great relevance. Generation of heat inside electronic devices must be dissipated quickly and the problem can be approached by building efficient conductivity paths in these devices [4]. Volume to Point problems can also be found in many others engineering applications, such as heating, chemical cooling reaction, feed cooling, and increasing the thermal conductivity of the energy storage media through carbon fiber [5]. In addition, VP problems have applications in a wide variety of different sciences such as biology, economics, and urban transport [4,9,10].

Constructal Theory (CT) is becoming a powerful methodology, being understood by many researches as a generation of the tendency of all systems (animate and inanimate) of nature to evolve, searching for shapes (paths) for flows (heat, fluid, people, etc.) that offer less ”resistance” [4]. For being a general theory, it can be applied in a large number of domains, where there is some kind of evolution. Thus, this theory has been used in solving problems of biology, physics, social organizations, technological evolution, sustainability and engineering, among others [4,11]. The strategy that emerged from seeing and applying the CT in basic flow configurations is called Constructal Design. When talking about the Constructal Theory and its usefulness for the optimization of engineering designs, it is necessary to think about the important idea spread by Ref. [11] which expresses that ”the engineered world we have built so that we can move more easily does not copy any part of the natural design; it is a manifestation of it. That said, once we know the principle, we can use it to improve our designs”.

The evolutionary computation techniques, and in particular the Genetic Algorithm (GA), have been used in recent years to search for optimal forms in many problems related to thermal area. It is an heuristic that allows to arrive at good (almost optimal) solutions, with a reasonable computational cost appropriate to the application needs [12].

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Studies have been found to approach VP problems using different methodologies, theories and, in some cases, an appropriate algorithm is proposed. Many works have used Constructal Theory to determine close to optimal conductivity pathways by developing "growth" algorithms were final form is deterministically obtained. Construction starts with the creation of a reference Basic Form (BF) and evolves to its final design either by building blocks of these high conductive BF material ([1] [2] [3]), by adding new elements of this BF [6] or rearranging these BF [13]. In all cases, paths with tree-like forms were obtained.

Ways to improve design of conductive pathways of a predefined form have been studied by many authors. In these works, optimal relationship among dimensions (height to length ratio, for example) are obtained by employing an optimization algorithm (exhaustive search in most of the cases) to solve similar problems subjected to different external conditions. An example of this kind of CT application is the work of [8] on which CT is applied to discover the architecture that maximizes X-shaped pathways performance in cooling heat-generating bodies, which was extended in Ref. [7] to discover the best configuration for a non-uniform X-shaped pathway. Constructal Theory was also applied in Ref. [14] for an I-shaped pathways in thermal contact applications, and in Ref. [15] for various configurations for a "+"-shaped pathway. Moreover, the works presented by Refs. [16] and [17] employed CT to numerically evaluate the geometric configuration of a T-shaped conductive pathway and considered the numerical optimization of a T-shaped assembly of fins cooling a cylindrical solid body, respectively.

Another similar application of the CT relies on the isothermal cavity problem. Cavities with specific forms that penetrate into a solid conducting wall have been studied using CT. Previously mentioned work of [18] studied not only the geometry of a X-shaped conductive pathway, but also of an isothermal cavity with the same shape. Examples of interesting problems on the subject can be seen in Ref. [19] who optimized a rectangular cavity and a T-shaped cavity, in Ref. [20] who did it for H-shaped cavities and reproduced results of C and T-shaped cavities, aiming to compare their performance, in Ref. [21] that optimized the geometry of T-Y-shaped cavity and [22] who did it for a single T-shaped cavity. In addition [23], and [24] used GA to optimize a Y-shaped cavity. A work that built cavities in an evolutionary way was developed by Ref. [25] who proposed an algorithm to create the cavity from the removal of small elements of material, called Elemental Constructal (EC) by the authors. On the other hand, it is also important to highlight the work developed by Ref. [5] who, despite not using the CT, address the VP problem and developed a new methodology using Simulated Annealing and GA.

This work proposes two new methodologies to determine close to optimum configurations for a solid (2D plate) constructed with two types of materials and that is being submitted to a heat source. That is, to generate a strategy for finding appropriate positions for the high and low thermal conductivity materials with which the plate is constructed so as to minimize the global thermal resistance (also called "maximum temperature" or "excess of temperature" in literature) occurring in the computational domain.

Therefore, in this work the Constructal Theory is used, together with the GA technique, to develop a strategy that allows to get close to optimal geometries formed by the material of high conductivity inside the plate.

2. Problem definition

Consider the two-dimensional conducting plate shown in Fig. 1, with height $H\,[m]$ and length $L\,[m]$, subjected to a uniform volumetric heat generation rate ($q'''\,[W/m^3]$) given by

$$q''' = \frac{q_0}{(A - A_p)W} \tag{1}$$

where q_0 is the heat generation rate $[W]$, A is the area $[m^2]$ and A_p is the area occupied by high conductivity material $[m^2]$.

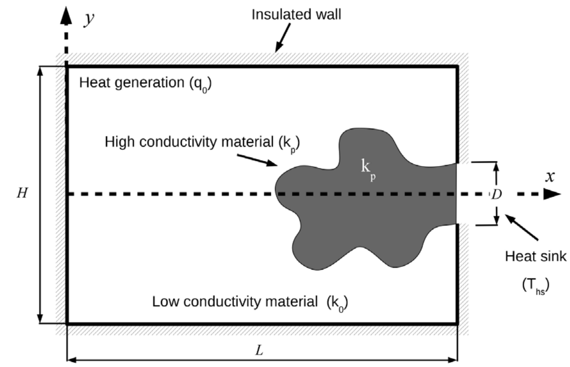

Fig. 1. Problem description.

The heat generated is removed through a heat sink of height $D\,[m]$ located in right wall of the plate at temperature $T_{hs}\,[m]$. All other surfaces of the heat generating plate are perfectly insulated. The plate is constructed using materials of low (k_0) and high (k_p) thermal conductivity. In addition, Fig. 1 shows a possible configuration for the problem where the gray region represents the high conductivity material and the white region the low conductivity material inside the plate. The problem is defined as pure conduction, steady state in a 2D computing domain.

The equations that govern the problem are given by:

$$k_0\frac{\partial^2 T}{\partial x^2} + k_0\frac{\partial^2 T}{\partial y^2} + \frac{q_0}{(A - A_p)W} = 0 \tag{2}$$

for the region with low thermal conductivity material, and

$$k_p\frac{\partial^2 T}{\partial x^2} + k_p\frac{\partial^2 T}{\partial y^2} = 0 \tag{3}$$

for the region with high thermal conductivity material.

In Eqs. (1)–(3), T is the temperature $[K]$, x and y are Cartesian coordinates $[m]$, W is the thickness of the plate (equal to 1) $[m]$ and the subscripts 0 and p indicate the low and high conductivity, respectively.

The following dimensionless variables are defined:

$$(\tilde{x}, \tilde{y}, \tilde{H}, \tilde{L}, \tilde{D}, \tilde{A}) = \frac{(x, y, H, L, A)}{A^{(1/2)}},$$
$$\theta = \frac{(T - T_{hs})W}{q_0/k_0}, \quad \phi = \frac{A_p}{A} \tag{4}$$

where ϕ is the high conductivity material area fraction and θ is the thermal resistance.

Analysis that allows to calculate the dimensionless thermal resistance, as a function of the geometry, consists of numerically solving Eq. (5) along the low conductivity region and Eq. (6) throughout the region of high conductivity:

$$\frac{\partial^2\theta}{\partial\tilde{x}^2} + \frac{\partial^2\theta}{\partial\tilde{y}^2} + \frac{1}{1 - \phi} = 0 \tag{5}$$

$$\frac{k_p}{k_0}\frac{\partial^2\theta}{\partial\tilde{x}^2} + \frac{k_p}{k_0}\frac{\partial^2\theta}{\partial\tilde{y}^2} = 0 \tag{6}$$

The maximum dimensionless thermal resistance is defined by:

$$\theta_{max} = \frac{(T_{max} - T_{hs})W}{q_0/k_0} \tag{7}$$

where subscript max indicates maximum.

The outer surfaces are insulated while prescribed temperature is set at the heat sink. Problem boundary conditions are:

$$\frac{\partial\theta}{\partial\tilde{x}} = 0 \quad \text{at} \quad \tilde{x} = 0 \quad \text{and} \quad -\frac{\tilde{H}}{2} \le \tilde{y} \le \frac{\tilde{H}}{2} \tag{8}$$