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Criteria for accurate measurement of thermal diffusivity of solids using the Angstrom method

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ABSTRACT

This paper presents criteria for accurately measuring the thermal diffusivity of a solid material using the Ångström method. In this technique, a periodic heat load is supplied at one end of the sample and temperatures are measured at different locations along the sample. The thermal diffusivity is estimated from the measured amplitude and phase of the temperature oscillations at various locations along the sample. Criteria are presented for important measurement parameters namely as periodic heat load power, waveform and frequency. Since more than two sensors are used, the accuracy is further improved since the linearity of the phase and amplitude data can be verified. We experimentally establish that a sinusoidal input heat load with a frequency as defined by the criteria established in this paper results in improved measurement accuracy. Based on these criteria, the experiment was performed with a range of materials spanning low to high thermal diffusivities namely Teflon, G10, Titanium Alloy (Ti6Al4V), Stainless steel (SS316) and Aluminium Alloy (Al6061-T6). The results indicate that the measured thermal diffusivity values deviate from the literature data at room temperature by less than 2.1%.

1. Introduction

The Ångström or thermal wave technique is often used to measure the thermal diffusivity of solid materials [1–3]. This paper presents, and experimentally validates, criteria for accurate measurements using the Ångström method over a wide range of thermal diffusivity values.

The thermal diffusivity (α) of a material is a measure of how fast a thermal disturbance propagates through it and is related to the density, thermal conductivity and specific heat by the relation $\alpha = \kappa/(\rho C_p)$. The thermal diffusivity of a specimen is normally directly measured by utilizing *transient methods* that utilize the measured transient temperature response to a time varying heat source. As stated in Tye [2], transient methods are classified into transitory and periodic temperature methods. In transitory methods the thermal diffusivity is estimated from the sample temperature response to a *sudden* change in input heat. A well-known example is the Flash method [4]. In the periodic temperature method, thermal diffusivity is estimated from the sample response to a *periodic* (time varying) heat input. Examples include the 3ω-method [5] and the Ångström method [3].

The Ångström method was first demonstrated in 1861 [3]. It is used to measure the thermal diffusivity of average to good thermally conductive materials. In the original experimental set-up, a rod is heated at one end, by periodically switching the heat load on and off, and its temperature is measured at two locations along the length. Thermal diffusivity of the rod is then estimated with the formula

$$\alpha = \frac{L^2}{2t\ \ln\frac{\theta_1}{\theta_2}}$$

Note that t is time that the thermal wave takes to transverse the distance L between two temperature sensors located along the sample.θ_1 and θ_2 are the amplitudes of the temperature wave at the two sensor locations. This method is relatively easy to setup and gives accurate results even with a convective environment. However, to the knowledge of the authors, there are few studies for materials with thermal diffusivity below $1 \times 10^{-6}\,m^2/s$ [6–8]. Additionally, there are no clear criteria in the open literature for determining the voltage waveform, frequency and heater power.

This paper discusses a number of approaches to improve the accuracy and range of the Ångström method so as to achieve accurate measurements for materials with a broader range of thermally conductivities. In this approach instead of two sensors (as stated in original experimental set-up), multiple sensors are used along the sample. This methodology not only improves the accuracy of the result but also gives better representation of the thermal wave behaviour propagating along the sample. The thermal diffusivity is estimated from the phase lag and

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Nomenclature

Latin symbols

A	Cross-sectional area of the sample [m^2]
A_n	Amplitude of the nth mode in the temperature signal
C_p	Specific heat [J/Kg K]
f	Frequency of periodic heating [Hz]
h	Heat transfer coefficient [W/m^{2}K]
H	$hp/\rho C_pA$
p	perimeter of the sample ($2\pi r$) [m]
Q	Heat Supply [Watts]
r	Radius of the sample[m]
R	Resistance [Ω]
T_m	Mean temperature of the sample [K]
T_∞	Surrounding temperature [K]
T	Time [s]

Greek symbols

ω	Angular Frequency of the wave signal
Φ_n	Phase of the nth mode in the temperature signal
α	Thermal diffusivity [m^2/s]
ε	Emittance of the sample
σ	Stefan-Boltzmann constant [W/m^{2}K^4]
ρ	Density [kg/m^3]
κ	Thermal Conductivity [W/m K]
θ	Amplitudes of the temperature wave[K]

Abbreviations

TW	Transient Temperature Wave
FFT	Fast Fourier Transform
Bi	Biot Number ($h\,r/\kappa$)

amplitude decay of the temperature response at various locations along the specimen. Frequency, waveform and power criteria for the thermal diffusivity measurement are also presented.

2. Theory

In the Ångström technique, a periodic heat load is applied at one end of a solid rod, with the entire length of the rod exposed to the surroundings. A temperature wave propagates along the rod, in which the phase gets shifted and the amplitude of the oscillations decreases. The temperature of this wave is measured as a function of time at various locations along the axis of the rod.

In the current analysis, it is assumed that the transverse temperature gradients are negligible (note that Bi < 0.1 with respect to the specimen radius for all samples considered in this study [9]) in a long thin rod shaped specimen (i.e. fin approximation [1]), hence the transient *one-dimensional* equation for heat conduction in the specimen is given by

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} + \frac{hp}{\kappa A}(T - T_\infty) \tag{1}$$

where, p and A are the cross section perimeter ($p = 2\pi r$) and area of the cylindrical rod sample, respectively. Since one end of the specimen is heated periodically with an angular frequency of ω ($\omega = 2\pi f$), the temperature amplitude at every location along the specimen will vary with this frequency and hence has the form

$$T(x,t) - T_m(x) = V(x)e^{i\omega t} \tag{2}$$

$V(x)$ is a *complex* position dependent quantity that describes the amplitude and phase of the resultant thermal wave. $T_m(x)$ is the mean temperature of the sample at a given location "x" and is obtained by solving the standard steady state fin equation [1,9]:

$$\frac{d^2T_m(x)}{dx^2} = m^2(T_m - T_\infty) \tag{3}$$

Note that

$$m^2 = \frac{hp}{\kappa A} = \frac{H}{\alpha} \tag{4}$$

where $h = h_{cov} + h_{rad,}$ where h_{cov} is the convective heat transfer coefficient and $h_{rad} = 4\varepsilon\sigma T_\infty^3$ is the radiation heat loss coefficient. Note that $H = hp/\rho C_pA$, ε is the emittance of the sample, σ is the Stefan-Boltzmann constant and T_∞ is the temperature of the surroundings.

Substituting Equation (2) in Equation (1)

$$\frac{d^2T_m(x)}{dx^2} + e^{i\omega t}\frac{d^2V(x)}{dx^2} = \frac{1}{\alpha}(i\omega)e^{i\omega t}V(x) + m^2[T_m - T_\infty + V(x)e^{i\omega t}] \tag{5}$$

Substituting Equation (3) in Equation (5) and rearranging

$$\frac{d^2V(x)}{dx^2} = \left[\frac{1}{\alpha}(i\omega) + m^2\right]V(x) \tag{6}$$

Let

$$\beta^2 = \left[\frac{i\omega}{\alpha} + m^2\right] \tag{7}$$

Thus Equation (6) has the form

$$\frac{d^2V(x)}{dx^2} = \beta^2 V(x) \tag{8}$$

The general solution of Equation (8) is

$$V(x) = C_1e^{\beta x} + C_2e^{-\beta x} \tag{9}$$

In Equation (9), C_1 is zero for a physically meaningful solution since the temperature cannot increase with distance in a fin. Substituting Equation (9) in Equation (2) the transient temperature amplitude is

$$T(x,t) - T_m(x) = (C_2e^{-\beta x})e^{i\omega t}$$

Since β is a complex number it can be represented as

$$\beta = z_1 + iz_2$$

z_1 and z_2 are respectively the real and imaginary components of β and $i = (-1)^{1/2}$. Substituting β in the above equation

$$T(x,t) - T_m(x) = C_2e^{-z_1x}e^{i(\omega t - z_2x)} \tag{10}$$

Equation (10) represents the transient temperature wave (TW) in the sample along the x-axis. Physically, $1/z_1$ is the length scale of the decay in the *amplitude* and z_2 is the *wave number*. Note that,

$$\beta^2 = (z_1 + iz_2)^2 = \left[m^2 + \frac{i\omega}{\alpha}\right] \tag{11}$$

Equating the real and imaginary parts respectively:

$$z_1^2 - z_2^2 = m^2 = \frac{H}{\alpha} \tag{12}$$

$$2z_1z_2 = \frac{\omega}{\alpha}$$

Solving Equation (12) for z_1 and z_2 yields

$$z_2 = \left[\frac{\sqrt{H^2 + \omega^2} - H}{2\alpha}\right]^{\frac{1}{2}}$$