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An extended approach of a Kalman smoothing technique applied to a transient nonlinear two-dimensional inverse heat conduction problem

Nejla Gaaloul, Naouel Daouas*

Ecole Nationale d'Ingénieurs de Monastir, Unité de Métrologie et des Systèmes Energétiques, Département de Génie Energétique, Université de Monastir, Rue Ibn El Jazzar, 5019, Monastir, Tunisia

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ABSTRACT

An extended Kalman smoother (EKS) introducing the concept of future measurement information is developed to handle nonlinear inverse heat conduction problems. In the present study, the formulation of the EKS algorithm is generalized in the case of a two-dimensional problem in order to reconstruct a non-homogeneous heating condition on the front surface of a cylindrical sample. This leads to apply the new method in a multiple sensor case for the simultaneous reconstruction of several parameters at each time. The analyzed inverse problem is nonlinear due to the variability of the thermophysical properties with temperature and to the presence of radiation boundary conditions. Inverse estimation is successfully performed where temporal and spatial variations of the front surface heat flux are recovered based on non intrusive transient temperature measurements made on the back surface. Numerical experiments show that the use of an optimal number of future data greatly improves the solution of the EKS compared to the extended Kalman filter (EKF) estimates, where a noticeable reduction of time lag and sensitivity to measurement errors is observed. Inversion results show that the optimal number of future data, chosen on the basis of a minimum measure of the heat flux bias, depends on the modeling error, the measurement time step, the distance between the sensors and the front heated surface and the measurement noise level. The proposed algorithm is robust in recovering different heat flux profiles and provides for all the examined patterns symmetric and stable solutions that superposes well with the exact functions. The EKS formulation based on an augmented state vector allows to efficiently recover the front surface temperature simultaneously with the heat flux.

1. Introduction

The solution of an inverse heat conduction problem (IHCP) is defined as the reconstruction of unknown heat flux or temperature on the surface of a heat conducting body based on temperature measurements taken at interior or backside points [1]. This situation is encountered when a direct measurement of a surface condition is not possible, either because the surface is unsuitable for attaching a sensor (aggressive environment, rubbing, etc.), or because the presence of the sensor may impair the accuracy of the direct measurement on the surface.

The resolution of IHCPs concerns several applications including heating in internal combustion chambers [2] or in internal boilers [3], laser heating [4,5] and space vehicles heating at the reentry of the earth's atmosphere [6].

IHCPs are mathematically ill-posed, mainly because of their high sensitivity to random errors of measurements, which can result in considerable perturbations in the solution. Indeed, the nature of transient heat conduction in a solid is such that a disturbance on the surface is damped and lagged toward the interior. Inversely, when using interior point measurements as inputs for the inverse problem, the least measurement error will be amplified at the surface, causing large oscillations and instability in the estimated surface condition.

Special techniques are crucial to ensure a stable and correct estimation of the inverse solution. One of the most efficient techniques is the use of information at times ulterior to the estimation instant called future time measurements. This technique has been firstly developed by Beck [7] in a least-square sequential procedure denoted by the function specification method which greatly enhanced the stability of the inverse problem. Improved versions of the Beck's method have been proposed later [8–11]. Furthermore, Hensel and Hills [12] efficiently introduced the concept of future time measurements in the analysis of a linear IHCP based on a numerical procedure without iterations. In turn, Raynaud and Bransier [13] developed a space marching method based on future time data. Recently, their method has been combined with the

* Corresponding author.
E-mail address: naou.daouas@gnet.tn (N. Daouas).

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Nomenclature

B	bias
C	volumetric heat capacity (W/m^3.K)
e	cylinder thickness (m)
EKF	extended Kalman filter
EKS	extended Kalman smoother
h	convection heat transfer coefficient (W/m^2.K)
H	observation matrix
I	identity matrix
IHCP	inverse heat conduction problem
K	filter gain
N	total number of nodes
n_f	number of future measurements
n_r	number of nodes in the radial direction
n_t	total number of time steps
n_z	number of nodes in the axial direction
p	probability
P	covariance matrix of estimation error
Q	covariance matrix of model error
q_0	maximum value of surface heat flux (W/m^2)
q_s	surface heat flux (W/m^2)
r	radial coordinate (m), noise of model error (W/m^2)
R	cylinder radius (m)
R	covariance matrix of measurement error
t	time (s)
T	temperature (K)
T_a	ambient temperature (K)
x	state vector
X	augmented state vector
x*	reference state vector
y	measurement
y	vector of measurements
z	axial coordinate (m)
z_m	axial location of the sensors (m)

Greek symbols

α	thermal diffusivity (m^2/s)
Δr	space step in the radial direction (m)
Δt	inverse time step (s)
Δt_d	direct time step (s)
Δz	space step in the axial direction (m)
ε	emissivity
λ	thermal conductivity (W/m.K)
σ	Stefan-Boltzmann constant (W/m^2.K^4)
σ_m	standard deviation of measurement noise (K)
σ_q	standard deviation of modeling error (W/m^2)

Subscripts

i	refers to axial position
j	refers to radial position
k	refers to time
k/n	refers to (n-k) future times
m	refers to measurement

Superscripts

ˆ	estimation value
–	prediction value
t	transposition operator

function specification method and applied in an on-line procedure for the solution of a nonlinear IHCP [14].

The Kalman filter (KF) is a recursive stochastic algorithm whose performance has been widely demonstrated from the state estimation for different types of linear systems. In its estimation procedure, the KF efficiently extracts information from observations of the system outputs and is powerful in handling the random character of measurement noise when it is Gaussian and white. The original version of the KF applies to linear dynamic systems where the state equation is linear and the measurements are also related to the state by a linear equation [15]. Then, an extension of the basic KF has been introduced [16] in order to deal with nonlinear systems widely encountered in practice. The resulting filter is referred to as the extended Kalman filter (EKF).

In the literature, the majority of papers dealing with the solution of inverse heat conduction problems for the recovering of a heat flux, a temperature or a heat source history are limited to the application of the linear KF coupled with the recursive-least square algorithm (RLSA) [17–21]. The same technique (KF-RLSE) has been also applied for the solution of inverse radiation-conduction problems [22,23]. Little work has been done using the EKF. Wang et al. [24] and Chen et al. [25] combined the EKF with the RLSA to recover a transient heat source. The same combination (EKF-RLSA) has been adopted by Jang and Cheng [26] for the estimation of a time-varying heat source generated by a semiconductor electronic device. Only Daouas and Radhouani [27] proposed an approach of the EKF allowing a simultaneous correction of the process state (temperatures) and the surface condition history (transient heat flux) by adopting an augmented state vector, where both the process state and the unknown parameters are involved. This approach has the advantage of reducing the two-stage procedure of the EKF-RLSA algorithm to a one part calculation based on the sequential EKF equations. Moreover, it allows avoiding some additional initializations needed in the RLSA, on the one hand, and the crucial tuning of the adaptive weighing factor which controls the tracking of the surface condition transient variation in the RLSA, on the other hand.

The augmented state concept has been successfully applied for the solution of a nonlinear IHCP with a short computation time [27]. With an appropriate adjustment of the EKF tunable parameters, the stability of the estimation procedure has been ensured in spite of the presence of real noisy data. However, the recovered surface heat flux showed a time-lag, compared to the exact solution, which depends notably on the measurement location. Based on these results, the same authors attempted later to introduce the concept of future measurement information in the EKF algorithm [28]. They proposed a new formulation of a Kalman smoother technique, extended to a nonlinear model, and applied it to the solution of a transient one-dimensional IHCP including the reconstruction of time-varying heat flux and temperature at the surface of a cylindrical heat conducting solid. This extended Kalman smoother (EKS) greatly reduced the time-lag of the solution and also improved its stability. The EKS technique has been also tested in the presence of a real set of experimental noisy temperature measurements [29] and provided symmetrical and stable estimations of the histories of heat flux and temperature at the surface of a heat conducting cylinder. Compared to the reference function specification method proposed by Beck [7], the EKS scheme has some advantages related to its statistical approach. This allows the EKS algorithm to describe the stochastic structure of experimental measurements and to handle the random character of the measurement noise. The formulation of the EKS, takes into account the incertitudes in the model and in the initial state through their associated covariance matrices. Based on the concept of the augmented state vector, the EKS is able to smooth errors affecting both the initial temperature distribution and the surface condition.

In a similar context, Wan et al. [30] developed an unscented Kalman smoother combining the unscented Kalman filter and the Rauch Tung Striebel (RTS) smoother in order to solve a one-dimensional nonlinear