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Numerical simulation of coupled fluid flow and heat transfer with phase change using the Finite Pointset Method

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ABSTRACT

The purpose of this work is to carry out a meshfree implementation for the numerical simulation of two-dimensional transient incompressible flows coupled with heat transfer where phase change is present. The Finite Pointset Method is applied in order to solve the involved partial differential equations where the corresponding classical or strong formulation is directly used instead of the corresponding weak form as needed for some other meshfree approaches. The incorporation of the boundary conditions is done in a direct and simple manner. The simplicity and efficiency of this numerical method are demonstrated on several two-dimensional benchmark problems where solidification and melting processes occurs.

1. Introduction

Fluid flow coupled with heat transfer where phase change takes place arises in many engineering applications. Mesh-based numerical methods for partial differential equations have been used for solving such problems. However, it is very difficult to model phase transitions and discontinuities with mesh-based techniques since the interface location does not coincide with the original nodal lines during the problem evolution. It is very difficult for this kind of methods to simulate material fragmentation since most of them are based on continuum mechanics in which the elements cannot be broken. Therefore, a false representation of the fragments paths could be produced. The remeshing approaches has been proposed in mesh-based methods to overcome the problems above and to simulate problems where phase changes occurs, which are both computational and economically expensive. Recently meshfree or meshless methods have been developed as an alternative to overcome part of the difficulties arising when mesh-based methods are used and these are classified in two main groups according to the type of equations on which they are based [1,2].

Some meshfree methods are based on the weak-form of the corresponding partial differential equations and they are characterized by being stable and accurate, therefore they naturally satisfy the imposed Neumann boundary conditions. These methods are computationally expensive since the numerical integration is mandatory. Furthermore, these methods require local or global meshes for integrating the derived matrix system from the weak-form on the problem domain which makes them not completely meshfree. The most common examples of such methods include the Element Free Galerkin Method (EFG), the Reproducing Kernel Particle Method (RKPM), the Diffuse Element Method (DEM), the Meshless Local Petrov-Galerkin Method (MLPG), Meshless Boundary Element Method (MBEM), Meshless Finite Volume Particle Method (MFVM) and the Natural Element Method (NEM). There are other meshfree methods which are based on the strong-form of the corresponding partial differential equations and they are characterized by being truly meshfree since they do not require any kind of meshing during the solving process, moreover they are easy to implement and computationally efficient. Nonetheless, many of them are unstable and less precise than weak-form methods when Neumann boundary conditions are involved. The most common examples of these kind of methods include the Smoothed Particle Hydrodynamics (SPH), Finite Pointset Method (FPM), Finite Point Method and the Radial Basis Function collocation Method (RBFCM) [1–8].

Different meshfree methods have been applied in order to numerically solve for fluid flow coupled with heat transfer including phase change, Vertnik and collegues develop the local radial basis function collocation method to solve non-linear convective-diffusive transport phenomena problems with non-linear material properties with phase change and solve a billet casting problem with simultaneous material and interphase moving boundaries by an upgraded version of the LRBFCM meshless method [9,10], Zhang et al. applied the classical finite point method for solidification modelling in continuous casting [11], Kosec studied meshfree local radial basis function collocation method for coupled heat transfer and fluid flow problems, Fang et al. used an improved SPH model for droplet spreading and solidification

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Nomenclature

J	Functional [·]
M	Auxiliary matrix [·]
P	Differences matrix [·]
P_{xy}	Differences matrix [·]
Q	Auxiliary matrix [·]
T	Fluid temperature [°C]
T_c	Cold/reference temperature [°C]
T_h	Hot temperature [°C]
T_l	Liquid temperature [°C]
T_m	Melting temperature [°C]
T_s	Solid temperature [°C]
W	Weight matrix [·]
$b_k(\mathbf{x})$	Coefficients in Taylor series [·]
$\mathbf{b}$	Unknowns vector [·][J kg^{-1}°C^{-1}]
c	Volumetric heat capacity
c_l	Specific heat capacity for liquid region [J kg^{-1}°C^{-1}]
c_s	Specific heat capacity for solid region [J kg^{-1}°C^{-1}]
$\mathbf{e}$	Truncation error vector [·]
f	Arbitrary function value [·]
$\tilde{f}$	Approximated function value [·]
$\mathbf{f}$	Function value vector [·]
$\mathbf{f}_b$	Distributed body force [m s^{-2}]
$\mathbf{g}$	Gravitational acceleration vector [m s^{-2}]
h	Smoothing length in w [m]
$h_{i,k}$	Spatial differences [·]
h_f	Latent heat of fusion [Jkg^{-1}]
k	Thermal conductivity [W m^{-1}°K^{-1}]
k_0	Thermal conductivity at reference temperature [W m^{-1}°K^{-1}]
k_s	Thermal conductivity for solid phase [W m^{-1}°K^{-1}]
k_l	Thermal conductivity for liquid phase [W m^{-1}°K^{-1}]
$\mathbf{n}$	Boundary normal vector [·]
p	Flow pressure [Pa]
$p_k(\bar{\mathbf{x}})$	Linear independent functions [·]
t_f	Total simulation time [s]
$\mathbf{v}$	Flow velocity vector [m s^{-1}]
$\mathbf{v}_0$	Initial velocity [m s^{-1}]
$\tilde{\mathbf{v}}$	Temporal flow velocity [m s^{-1}]
w	Weight function [·]
$\mathbf{x}$	Arbitrary fluid point [m]
$\mathbf{x}^k$	Particle position at k-th iteration [m]
$\mathbf{x}_i$	i-th particle position [m]
Γ_d	Dirichlet boundary [·]
Γ_n	Neumann boundary [·]
Φ	Shape function [·]
Ω	A given fluid domain [·]
β	Coefficient of thermal expansion [°C^{-1}]
γ	Weight function parameter [·]
μ	Fluid dynamic viscosity [Pa s]
μ_m	Mushy zone viscosity [Pa s]
μ_s	Fluid dynamic viscosity for solid phase [Pa s]
μ_l	Fluid dynamic viscosity for liquid phase [Pa s]
ν	Fluid kinematic viscosity [m^{2}s^{-1}]
ρ	Fluid density [kg m^{-3}]
ρ_s	Fluid density for solid phase [kg m^{-3}]
ρ_l	Fluid density for liquid phase[kg m^{-3}]
$\partial\Omega$	Boundary of fluid domain [·]
$\frac{D}{Dt}$	Material derivative [s^{-1}]
∇	Gradient operator [m^{-1}]
Δ	Laplace operator [m^{-2}]
Δt	Time step [s]
Δx_i	Spatial differences [m]
Δy_i	Spatial differences [m]
Δz_i	Spatial differences [m]

simulation [12], Yang and He solved heat transfer with phase change using the element-free Galerkin method in combination with a smoothed effective heat capacity model [13], Thakur et al. used the meshless local Petrov-Galerkin method to solve phase change problems [14], Singh and Bhargava used an hybrid FEM/EFGM technique for the numerical simulation of a phase transition problem with natural convection [15], Li et al. applied the Lattice Boltzmann method for melting problems [16], Farrokhpanah et al. proposed a new SPH fomulation for modelling transient heat conduction with phase change [17], Dehghan and Najafi studied some high order mesh-based and meshless methods for non-classical one-dimensional two-phase Stefan problem [18], the extended finite element method (XFEM) has been applied by Stapór for two dimensional simulation of solidification processes in materials with thermo-dependent properties [19], Karagiannakis et al. used the meshless local Petrov-Galerkin method for transient thermal conduction with spatiotemporally variable conductivity [20], the Element-free Galerkin formulation has been applied to transient heat transfer problems of direct chill casting processes in Ref. [21].

The Finite Pointset Method (FPM) is a Lagrangian truly meshless approach developed by Kuhnert in Ref. [22] at the Fraunhofer-Institut für Techno-und Wirtschaftsmathematik, in Kaiserslautern, Germany. FPM has shown to be far superior to traditional mesh-based methods and some other meshfree method for solving Partial Differential Equations governing complicated physical phenomena since it can overcome some of the problems in SPH formulation and in other strong-form meshfree methods, especially those related to the treatment of the boundary conditions [23–30]. The main motivation in this work is that in FPM nodes can be added or removed as needed since it uses a set of finite nodes scattered within a problem domain as well as on its boundaries which do not carry mass, this provides stability advantage to the method as well as a wider range of boundary conditions which are essential for phase change problems. Moreover and to authors's knowledge, the use of FPM for solving fluid flow coupled with heat transfer considering phase change has not been reported in the scientific literature, therefore the application of FPM in this context is proposed in this work.

The structure of the paper is as follows: section 2 introduces the governing equations, section 3 shortly describes the numerical scheme for solving the system of PDEs, section 4 presents the main ideas behind FPM followed by its discretization presented in section 5. The numerical results are reported in section 6 and finally some conclusions are given in last section.

2. Governing equations

The governing equations are the incompressible Navier-Stokes equations in a laminar regime coupled with the convective heat transfer equation. In this work the molten material will be considered to behave as Newtonian fluid. Thus, the governing equations read:

$$\frac{D\mathbf{v}}{Dt} = -\frac{1}{\rho}\nabla p + \nu\nabla^2\mathbf{v} + \mathbf{f_b} + \mathbf{g}\beta(T - T_c) \tag{1}$$

$$\nabla\cdot\mathbf{v} = 0 \tag{2}$$

$$\frac{DT}{Dt} = \frac{1}{c}\nabla(k\nabla T) \tag{3}$$

Suitable boundary conditions for flow dynamics should be carefully defined, therefore no-slip conditions for solid walls are considered, this imply all velocity components, should be prescribed. The mushy zone viscosity μ_m is a function of temperature and it is defined as follows: