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A model for the drag and heat transfer of spheres in the laminar regime at high temperature differences

N. Ellendt[a,b,*], A.M. Lumanglas[a], S. Imani Moqadam[a], L. Mädler[a,b]

[a] University of Bremen, Faculty of Production Engineering, Badgasteiner Straße 1, 28359, Bremen, Germany
[b] Leibniz Institute for Materials Engineering IWT, Badgasteiner Straße 3, 28359, Bremen, Germany

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ABSTRACT

Modern powder metallurgical processes such as additive manufacturing or metal injection molding require metal powders with specific properties, which are commonly produced by gas atomization processes. Modeling of cooling and solidification of molten metal droplets requires the knowledge of the droplet and particle motion and heat transfer. While there are correlations for drag and heat transfer of spheres at isothermal conditions or temperature differences between droplet and gas smaller than 200 K, knowledge for temperature differences as high as 1000 K is limited. In this work, we first critically review common correlations for the drag coefficient and Nusselt number of spheres and develop a computational model to solve the non-isothermal flow for such conditions. After validation, new correlations are benchmarked against computational experiments. While we could confirm a good agreement for the Nusselt number correlations, we develop a novel temperature-dependent correction for the drag coefficient in the laminar regime. This correlation is based on data for $1 < \mathrm{Re} < 130$ for two different ideal gases.

1. Introduction

During the last 80 years, many studies have been conducted to determine the Nusselt number and drag coefficients of spheres and droplets for different conditions. Most of these studies have been performed at small temperature differences. In molten metal droplet processes such as droplet generators or melt atomization, high temperature differences of 1000 K or more can occur between the droplets and the ambient gas. Existing correlations are commonly used to model heat transfer in such processes although their range of validity is often exceeded with respect to temperature difference, Reynolds number, Prandtl number or thermophysical properties.

1.1. Drag correlations

The movement of a sphere falling free in a gas or within a gas flow is governed by the drag force F_D. The drag coefficient c_D is defined as

$$c_D = \frac{F_D}{A_r \frac{\rho_\infty}{2} u_\infty^2}, \tag{1}$$

where F_D is the drag force. This force is defined for a sphere with the reference area $A_r = \frac{\pi}{4} d^2$ being placed in a flow with the relative velocity u_∞ and the fluid density ρ_∞. In many applications, the flow conditions are known, therefore the resulting drag force can be calculated using the appropriate drag coefficient correlation. For a sphere, many correlations for the drag coefficient can be found in literature. For low Reynolds numbers $\mathrm{Re} < 1$, the flow is dominated by viscous forces and the drag coefficient can be derived as an exact solution as

$$c_D = 24 Re_\infty^{-1}. \tag{2}$$

For $1 < \mathrm{Re} < 800$ a well-known correlation outlined by Schiller & Naumann [1] is available:

$$c_D = 24 Re_\infty^{-1} (1 + 0.15 Re_\infty^{0.687}). \tag{3}$$

This correlation is commonly used in spray models [2,3] due to its simplicity and its reasonable agreement with experimental data.

However, the correlations mentioned above have been developed close to isothermal conditions ($T_S \approx T_\infty$). For $T_S \gg T_\infty$, the flow field is influenced by the heat transfer from the sphere to the ambient gas. For such conditions, it is not clear at which temperature (surface, film or free stream temperature) to calculate the gas properties and if an additional correction term for temperature may become necessary.

* Corresponding author. University of Bremen, Faculty of Production Engineering, Badgasteiner Straße 1, 28359, Bremen, Germany.
E-mail address: ellendt@iwt.uni-bremen.de (N. Ellendt).

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Nomenclature

Roman symbols

T	temperature
A	sphere surface
A_r	sphere reference area of drag force
p	pressure
Q	Heat flow rate
u	fluid velocity
u_∞	free stream velocity
h	convective heat transfer coefficient
d_S	sphere diameter
R^2	coefficient of determination
k	thermal conductivity

Greek symbols

ρ	density
η	dynamic viscosity
τ_w	shear stress, vertical component
ν	kinematic viscosity

Non-dimensional numbers

$Nu = \frac{hd}{k}$	Nusselt number
$Re = \frac{u_S d_S \rho}{\eta}$	Reynolds number
$Pr = \frac{\eta c_p}{k}$	Prandtl number
$Gr = \frac{g\beta(T_S - T_\infty)d_S^3}{\nu^2}$	Grashof number
$Ri = \frac{Gr}{Re^2}$	Richardson number
c_D	drag coefficient
$c_{D,SN}$	drag coefficient from Schiller-Naumann correlation
$c_{D,CFD}$	drag coefficient from flow model

Indices

S	property calculated at sphere surface temperature T_S
∞	property taken at ambient temperature T_∞
F	property taken at film temperature $T_F = \frac{T_S + T_\infty}{2}$

1.2. Nusselt number correlations

Whitaker [4] introduced correlations which match different heat transfer phenomena such as flow around spheres, cylinders or through packed beds. For the heat transfer of spheres, the correlation

$$Nu_\infty = 2 + \left(0.4Re_\infty^{\frac{1}{2}} + 0.06Re_\infty^{\frac{2}{3}}\right)Pr_\infty^{0.4}\left(\frac{\eta_\infty}{\eta_S}\right)^{1/4} \tag{4}$$

is valid for $3.5 \le Re_\infty \le 7.6 \times 10^4$, $0.71 \le Pr_\infty \le 380$ and $1.0 \le \left(\frac{\eta_\infty}{\eta_S}\right) \le 3.2$, with the Reynolds and Prandtl and Nusselt number being calculated at T_∞. To include a measure for the temperature difference, the ratio of dynamic viscosities at the surface Temperature T_S and ambient temperature T_∞ was introduced. However, to calculate a convective heat transfer coefficient from this Nusselt number, the use of the thermal conductivity at ambient temperature T_∞ is suggested which may introduce an error for $T_S \gg T_\infty$.

To avoid this error, Wiskel and Henein [5] introduced a correction to account for the temperature dependency of thermal conductivity in the boundary layer for increased temperature differences of $T_S - T_\infty = 800$ K and more:

$$Nu_S = \frac{2B}{k_s(m+1)}\frac{T_S^{m+1} - T_\infty^{m+1}}{T_S - T_\infty} + \left(0.4Re_S^{\frac{1}{2}} + 0.06Re_S^{\frac{2}{3}}\right)Pr_S^{0.4}\left(\frac{\eta_\infty}{\eta_S}\right)^{1/4}. \tag{5}$$

The authors validated their model producing aluminum droplets with impulse atomization and determined the solidification distance of the droplets experimentally and theoretically. However, one of their main assumptions is that impulse atomization is a single droplet process, while their process actually produces many droplets at a time [6]. Their assumed distance of 5 mm is the space between droplet clusters, while droplets within the clusters are much closer. Therefore, the assumption that there is no interaction between the droplets might not be valid. This also has consequences for the applied drag correlation. Also, it should be noted that the analytical solution for the minimal Nusselt number (Nu = 2) is no longer fulfilled in this correlation. The authors do not give a physical explanation on this issue.

Another common correlation was proposed by Ranz and Marshall [7]:

$$Nu_F = 2 + 0.6Re_F^{1/2}\,\mathrm{Pr}_F^{1/3} \tag{6}$$

with all gas properties being calculated at the film temperature $T_F = \frac{T_S + T_\infty}{2}$. It was developed for Reynolds numbers smaller than 200. This correlation has been obtained by measuring the temperature of water droplets through an air atmosphere for ambient temperatures with $85°C < T_\infty < 200$ °C.

Extensions of the Ranz-Marshall correlation to higher temperatures were presented for plasma spray applications. Fizdon et al. [8] measured the temperature of spherical particles with a diameter of 30–67 µm in a plasma flame at temperatures up to 8000 K and particle velocities of 540 m/s. They added a term to account for changes of density and viscosity:

$$Nu_F = 2 + 0.6Re_F^{1/2}\,Pr_F^{\frac{1}{3}}\left(\frac{\rho_\infty\eta_\infty}{\rho_S\eta_S}\right)^{0.6}. \tag{7}$$

Lee & Pfender [9] followed a similar approach for particles in a plasma flame. They modeled this with an additional correction for specific heat as

$$Nu_F = 2 + 0.6Re_F^{1/2}Pr_F^{\frac{1}{3}}\left(\frac{\rho_\infty\eta_\infty}{\rho_S\eta_S}\right)^{0.6}\left(\frac{c_{p,\infty}}{c_{p,S}}\right)^{0.38}. \tag{8}$$

However, it should be kept in mind that correlation (7) and (8) have been developed for plasma properties and high Mach number flows, where effects of compressibility can play a major role. Therefore, their applicability for ideal gases low Mach number flows is not necessarily given since the thermophysical properties show minima and maxima at different temperatures in the plasma range. This affects the Reynolds and Prandtl number strongly.

While the original correlation of Ranz and Marshall is valid only for Reynolds numbers lower than 200, Gnielinski [10] introduced a correlation which combines a laminar Nusselt number $\mathrm{Nu_{lam}}$ and a turbulent Nusselt number $\mathrm{Nu_{turb}}$. This combination extends the correlations' validity to Reynolds numbers of up to 10^6 for $0.7 < \mathrm{Pr} < 600$:

$$Nu_F = 2 + \sqrt{Nu_{lam}^2 + Nu_{turb}^2};\ Nu_{lam} = 0.644Re_F^{1/2}Pr_F^{1/3};\ \ Nu_{turb} = \frac{0.037Re_F^{0.8}Pr}{1 + 2.443Re_F^{-0.1}\left(Pr_F^{\frac{2}{3}} - 1\right)} \tag{9}$$

The correlation for the laminar Nusselt number is very similar to that found by Ranz and Marshall (first coefficient is 0.644 instead of 0.6). Yearling and Gould [11] introduced a term for the relative turbulence intensity σ_t to a correlation which is also similar to the equation obtained by Ranz and Marshall: