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Effects of wall heating on laminar mixed convection in a cylindrical enclosure with a rotating end wall

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ABSTRACT

Steady-state laminar mixed convection in a cylindrical enclosure has been numerically analysed for different values of Reynolds, Richardson and Prandtl numbers given by $500 \leq Re \leq 3000$, $0 \leq Ri \leq 1$ and $10 \leq Pr \leq 500$ respectively. The aspect ratio (*i.e.* height: radius = $AR = H/R$) of the cylindrical container is considered to be unity (*i.e.* $AR = H/R = 1$). The bottom and top covers of the cylindrical enclosure are kept at different temperatures ($T_C < T_H$), while the cylindrical surface is taken to be adiabatic. The simulations for rotating top and bottom cover configurations yield the same numerical values of the mean Nusselt number $\overline{Nu}$ when the thermal boundary conditions are kept unaltered. For this reason, only rotating top hot wall (*i.e.* C1 configuration) and rotating top cold wall (*i.e.* C2 configuration) have been considered for this analysis. The mean Nusselt number $\overline{Nu}$ has been found to assume higher values in the C2 configuration than in the C1 configuration. Moreover, it has been found that the variation of the mean Nusselt number with Richardson number in the C2 configuration is qualitatively different from that in the C1 configuration. The simulation data has been used to propose a correlation for $\overline{Nu}$ for the range of Re, Ri and Pr considered here for both C1 and C2 configurations. In addition to this, a regime diagram has been proposed for the C2 configuration in order to demarcate different flow regimes.

1. Introduction

The flow induced by rotating one of the covers of a cylindrical container has a wide range of applications (e.g. chemical processing, bio-chemical synthesis, polymer processing, food preparation, pharmacology). Mixed convection plays a vital role not only in heat transfer rate applications but also influences the mixing rate for low Reynolds number applications. Therefore, it is necessary to investigate heat transfer characteristics and flow structure in this configuration so that the rates of heat transfer and mixing can be optimised. However, heat transfer and mixing rates in this configuration depend on many parameters such as container geometry and the rotational speed of the cover. Two special cases of rotating flow problems are the flows on top of a rotating disk and inside an enclosure with a rotating end cover. Theodore von Karman pioneered the analysis of flows on top a rotating disk and such flows are commonly referred to as von-Karman flows, and an extensive review of such flows is provided in Ref. [1]. In addition to this, flows in cylindrical enclosures with a rotating cover have also been extensively analysed from various different viewpoints due to their relevance to a range of different engineering applications. Vogel [2,3], Ronnenberg [4] and Bertela and Gori [5] analysed fluid flows in cylindrical enclosures with a rotating end wall, and the findings of these studies [2–5] have subsequently been extended by Escudier [6] based on an experimental analysis where the criterion for vortex breakdown has been proposed in terms of aspect ratio H/R and Reynolds number $\Omega R^2/\nu$. Fujimura et al. [7] also experimentally investigated the flow generated by rotating end walls in a cylindrical container in which the top cover was rotated at a higher angular velocity than angular velocities at which the bottom and the side walls are rotated. The effects of the differential rotations of the container walls on the vortex breakdown have been found to be significant by Fujimura et al. [7]. Besides these experimental studies, several numerical investigations [8–14] analysed fluid flows in cylindrical enclosures with a rotating end wall. One of the fist numerical investigation was carried out by Lugt and Haussling [8], who focused on calculating the single recirculation bubble and verifying numerically the experimental data of Vogel [2] and Ronnenberg [4]. In addition to this, another pioneering numerical investigation was performed by Lopez [9] who was the first to calculate the full extent of the flows observed by Escudier [6]. Most investigations involving a rotating end wall focused on the investigation of flows in cylindrical containers until Pereira and Sousa [13], who investigated vortex breakdown generated by a rotating end wall within a conical

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Nomenclature

a	[-] Bridging function
AR	[-] Aspect ratio, ($AR = H/R$)
b	[-] Bridging function
c_p	[J/kgK] Specific heat at constant pressure
e_a	[-] Relative error
f_1	[-] General Function
g	[m/s^2] Gravitational acceleration
Gr	[-] Grashof number
h	[W/m^{2}K] Heat transfer coefficient
H	[m] Height of cylindrical enclosure
k	[W/mK] Thermal conductivity
k_0	[-] Correlation parameters
m_0	[-] Correlation parameters
Nu	[-] Nusselt number
$\overline{Nu}$	[-] Mean Nusselt number
Pr	[-] Prandtl number
q	[W/m^2] Heat flux
Q	[W] Heat transfer rate
R	[m] Radius of cylindrical enclosure
Ra	[-] Rayleigh number
Re	[-] Reynolds number
Ri	[-] Richardson number
T	[K] Temperature
U	(m/s) Characteristic velocity scales in radial direction
V	(m/s) Characteristic velocity scales in tangential direction
V_ϕ	[-] Non-dimensional swirl velocity, ($V_\phi = vH/\alpha$)

The Greek symbols

α	[m^2/s] Thermal diffusivity
β	[1/K] Coefficient of thermal expansion
$\dot{\gamma}$	[1/s] Shear rate
δ, δ_{th}	[m] Hydrodynamic and thermal boundary layer thickness
θ	[-] Non-dimensional temperature, ($\theta = (T\text{-}T_C)/(T_H\text{-}T_C)$)
μ	[Ns/m^2] Plastic viscosity
ν	[m^2/s] Kinematic viscosity
ρ	[kg/m^3] Density
τ	[N/m^2] Shear stress
Ω	[1/s) Angular velocity
ψ	[m^2/s] Stream function
Ψ	[-] Non-dimensional stream function, (Ψ= ψ/α)

Subscripts

adv	Advective
C	Cold wall
conv	Convective
diff	Diffusive
H	Hot wall
max	Maximum value
r	Radial direction
ref wall	Reference value Wall value
wf	Condition of the fluid in contact with the wall
z	Axial direction
ϕ	Tangential direction

container. The flow produced in a conical container by a rotating end wall has also been numerically analysed by Escudier et al. [14] who reported that vortex breakdown is suppressed beyond a certain angle of inclination of the sidewall for both convergent (increasing radius towards the rotating end wall) and divergent (decreasing radius towards the rotating end wall) geometries.

The analysis of heat transfer characteristics in cylindrical enclosures with a rotating cover received relatively limited attention [10–12]. Lee and Hyun [11] analysed the effects of Prandtl number on heat transfer rate in this configuration and revealed that Prandtl number has an important influence on the heat transfer characteristics and advective transport has been found to strengthen with increasing Prandtl number. Iwatsu [12] investigated the effects of Reynolds and Richardson numbers at $Pr = 1$, in the range of $100 \le Re \le 3000$, and $0 \le Ri \le 1$, on the flow pattern and heat transfer rate for swirling flows in cylindrical enclosures with an aspect ratio of unity (i.e. $AR = H/R = 1$), and a heated rotating top wall based on numerical simulations. The analysis by Iwatsu [12] revealed that advective (diffusive) transport strengthens (weakens) and accordingly, the mean Nusselt number increases with decreasing Richardson number.

There are four possible different configurations for the flows induced by rotating one of the covers of a cylindrical container depending on the boundary conditions, which are schematically shown in Fig. 1. Existing analyses on flow induced by the rotation of one of the end covers in a cylindrical enclosure have been summarised in Table 1, where the boundary conditions and the governing non-dimensional parameters for which these studies were conducted have also been summarised along with the nature of the investigation (i.e. whether it is experimental or numerical). It can be seen from Table 1, the majority of the existing analyses focused only on the flow structure, and there is no existing analysis in which the all possible configurations are investigated in terms of the heat transfer characteristics. For this purpose, the present study focuses on a detailed analysis of the heat transfer characteristics in cylindrical enclosures with a rotating end wall for different boundary conditions which are schematically shown in Fig. 1. A parametric analysis has been conducted to analyse the effects of Richardson, Reynolds and Prandtl numbers on heat and momentum transport for a range of Reynolds, Richardson, and Prandtl numbers (definitions are provided in Section 2) given by $500 \le Re \le 3000$, $0 \le Ri \le 1$ and $10 \le Pr \le 500$ respectively for an aspect ratio (height: radius) of unity (i.e. $AR = H/R = 1$). In this respect, the main objective of the present paper is to demonstrate the influences of Reynolds,

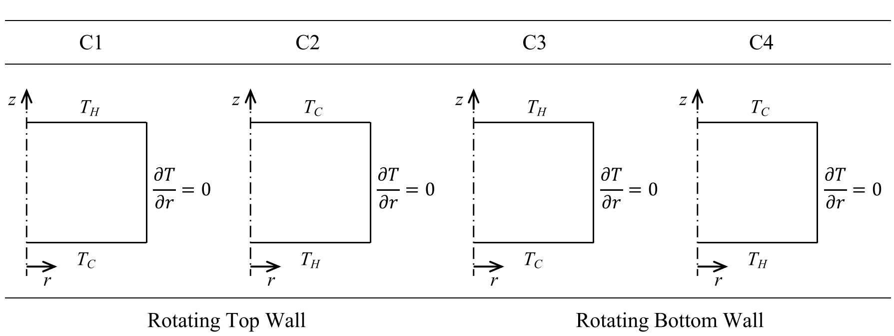

Fig. 1. Schematic diagrams of the simulation domain and different boundary condition cases where T_H and T_C are the hot and cold wall temperatures.