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Estimation of heat flux by using reduced model and the adjoint method. Application to a brake disc rotating

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ABSTRACT

In this study, an approach for inverse problem is proposed to estimate a time dependent heat flux on a braking system. This approach combines modal reduction and conjugate gradient method with adjoint method. A brake disc with variable velocity and thermal solicitations is treated. In a first step, Branch Eigenmodes Reduction Method is used to build a reduced model which allows reduction of computing time while conserving a satisfying precision on temperature field. In a second step, an inverse algorithm based on adjoint method is applied to recover time dependent heat flux from temperature calculation. Compared to Beck method, adjoint method gives accurate results without any regularization. The proposed approach enables to perform a quasi on-line estimation.

1. Introduction

The automobile brake disc is a major safety component. It undergoes during its operating phase many mechanical and thermal stresses. Experience shows that the main constraint is induced by high temperature gradient (between 0 °C and 800 °C) due to friction between the pad and the disc. In certain operating conditions, spaced areas where the temperature is higher appear, so-called *hot spots* [1]. Because of these strong thermal loads, the disc can undergo damages: cracks, apparition of hot-judder, vapor locking, brake fade …

Due to complex coupling of thermal and mechanical phenomena in a brake disc, numerical modeling of braking system to predict the damage can be complex, and leads to models (FEM for example) with very fine meshes. Computing time and memory problems appear very quickly.

Another difficulty arises from the poor knowledge of thermal solicitations, especially the part of the heat flux received by the pad and by the disc. To solve this kind of problem, inverse techniques are used.

The Beck method [2,3], which has the particularity of being sequential, has been used in many works including our previous work [4] and those of Meresse et al. [5], where the objective was to estimate the heat flux dissipated during the braking phase.

The adjoint method with Conjugate Gradient Method (CGM) [6,7] is an iterative technique based on the successive calculation of descent directions, minimizing a criterion taking into account all or part of data. This technique is increasingly widespread. It allows to solve different kind of inverse problem. For example, in Ref. [8] the CGM coupled with adjoint problem was used to estimate the moving heat source in machining process. Yang et al. [9] applied the inverse algorithm based on the CGM to estimate the heat flux in a brake disc system in 1D configuration. This technique has also been used in Ref. [10] to estimate frictional heat flux at the interface of two spaces during a sliding contact. Authors showed that accuracy of the inverse method depends on the exactitude of assumed known parameters. Recently Vergnaud et al. [11] used CGM to identify on-line heat fluxes provided by two mobile sources. They used temperature evolution at ten sensors placed around the trajectories of mobile sources. It appears from this study that the limitation of this method to achieve real-time estimation with classical FEM model is the computing time: this iterative procedure requires many runs of FEM model, involving time of resolution of inverse problem unbearably long. Numerical model was obtained with Comsol software and the number of degrees of freedom was 4749, which is relatively low compared to other coupled physical problems. To achieve quasi-online identification, authors propose several strategies and adaptation of CGM on a sliding-time window.

To avoid costly computation time, model reduction offers a very attractive alternative, particularly to study phenomena in real-time. The aim of reduced model is to reproduce the behavior of the FEM model (also called detailed model) with satisfying accuracy and computational time. Many studies are devoted to model reduction theme for different application areas such as fluid mechanics, heat transfer, and mechanics.

Among reduction methods that have demonstrated their efficiency in solving inverse problems, to name a few, one can cite the Proper

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Orthogonal Decomposition with Galerkin projection (POD-G), the Modal Identification Method (MIM) and the Branch Eigenmodes Reduction Method (BERM).

A POD-G is the well known reduction method in fluid mechanics community, also known as Karhunen-Loève Galerkin method. It was used in Ref. [12] to build a reduced model to perform an estimation of time varying heat sources in a two-dimensional non-linear heat conduction problem. The authors showed how the reduced model leads to a drastic reduction of CPU time while preserving accurate estimation compared to a conventional model: only 6 s are required for the estimation by using reduced model, whereas it takes 15 min with conventional model. In other study [13] Karhunen-Loève Galerkin method was used to investigate an inverse natural convection problem. The aim was to estimate the unknown strength of a time-dependent heat source from temperature observation within the flow.

The Modal Identification Method MIM [14–16] is based on the state representation under modal form. The reduced model is identified through the resolution of an optimization problem of parameter estimation. It has been employed in many configurations such as fluid mechanics [15], diffusion-convection problem [16] or purely diffusive heat transfer [14,17].

In Ref. [18], MIM was implemented with Beck method to estimate the time dependent strengths of two sources from measured temperature at some points. It has been also used in Ref. [19] to estimate the time varying boundary conditions in 3D case. In other studies [20,21] feedback optimal control applications using a reduced model based on the MIM and applied to a convection-diffusion problem were presented.

There are several other approaches such as rational Krylov subspace methods [22,23]. Indeed the rational Krylov space is recognized as a powerful tool in model order reduction techniques for linear dynamical systems.

The aim here is not to make a deep review of state of the art of model reduction, the reader interested in reviews on reduction techniques can refer to [24,25] for instance.

The approach adopted in this paper is the Branch Eigenmodes Reduction Method (BERM). It consists of solving a specific spectral problem called Branch problem. BERM is a powerful technique enabling to obtain reduced basis to handle non-linearities, especially in boundary conditions or time-varying parameters [26–29].

In previous work [4], the interest of working with reduced models for solving inverse problems has been established, especially when they were characterized by a complex geometry requiring a large number of nodes and/or a real time identification target. The treated application was a brake disc in two-dimensional representation, rotating at a variable velocity. The objective was to identify the heat flux dissipated at the pad-disc interface by using Beck method. The present work deals with the interest of CGM combined with adjoint problem to estimate a dissipated heat flux.

The paper is structured as follows:

- Section 2 describes the studied system and its modeling. Some simulation results are also presented.
- Section 3 is dedicated to the modal reduction and its numerical comparison between the detailed model and the reduced model.
- Section 4 presents the techniques used to solve the inverse problem.
- Section 5 summarizes the results of heat flux estimation obtained by both Beck method and CGM.
- Eventually, section 6 is devoted to conclusions and extensions of the presented work.

2. The studied system and its modeling

2.1. Physical system

A brake disc in rotation with variable rotation frequency $\omega(t)$ is considered. During braking phase the disc receives a time-dependent heat flux on the zone of friction with the brake pads Ω_1. The flux density φ [W.m^{-2}] dissipated by friction is not uniform but varies linearly with the velocity thus with the radius.

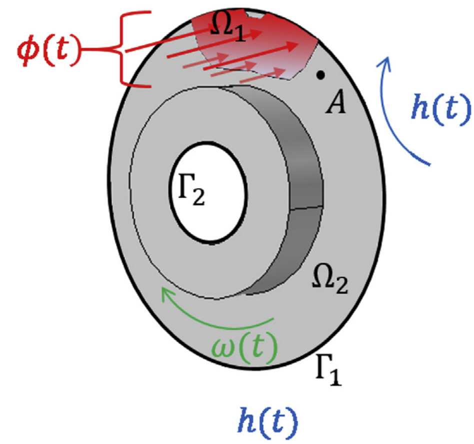

Fig. 1. Geometry of problem.

The disc has a diameter $D = 26.5$ cm and a thickness $e = 8$ mm, and is characterized by a thermal conductivity $k = 50$ W.m^{-1}.K^{-1} and volume specific heat capacity $c = 3.66\ 10^6$ J.m^{-3}.K^{-1}. With these characteristics, the Biot number corresponding to the most unfavourable case has as a value $Bi = 0.018 \ll 1$. It is then possible to neglect the thermal gradient in the thickness e of the disc. A thermal problem of shell-type, which is described in the following section, is thus obtained (see Fig. 1).

In this study the initial temperature is considered to be uniform and equal to the ambient temperature $T_\infty = 0$°C.

The various time-varying parameters, namely the rotation frequency $\omega(t)$, the coefficient of the convective heat exchange $h(t)$ and the heat flux dissipated by friction $\phi(t)$ are expressed according to their maximal values, respectively ω_m, h_m and ϕ_m:

$$\omega(t) = \omega_u(t)\,\omega_m \quad \text{and} \quad h(t) = h_u(t)\,h_m \tag{1}$$

and

$$\phi(t) = \int_{\Omega_1} \varphi(M, t)\mathrm{d}\Omega = \phi_u(t)\int_{\Omega_1} \varphi_m(r)\mathrm{d}\Omega = \phi_u(t) \times \phi_m \tag{2}$$

The time evolution of $\omega_u(t)$, $h_u(t)$ and $\phi_u(t)$ are plotted in Fig. 2, and corresponding maximal values are: $\omega_m = 2\pi$ rad.s^{-1}, $h_m = 110$ W.m^{-2}.K^{-1} and $\phi_m = 600$ W.

2.2. Model

The physical system is modeled by the following equations:

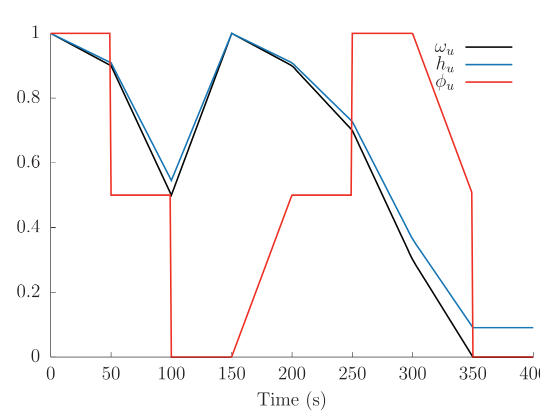

Fig. 2. Evolution of test case braking.