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Numerical study of laminar natural convection heat transfer from a hemisphere with adiabatic plane and isothermal hemispherical surface

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ABSTRACT

This work numerically studied laminar natural convection heat transfer from a hemisphere with the downward adiabatic flat and isothermal hemispherical surface in the range of Grashof numbers $10 \le Gr \le 10^6$ and for Prandtl number of 0.72. The comprehensive results consisting of the streamlines and isothermal contours, temperature and velocity profiles, as well as local and average Nusselt numbers, have been presented with different Grashof numbers, and compared with the case of the hemisphere that all surfaces are isothermal. They reveal that at low Grashof numbers, although the plane is adiabatic, there exists the temperature boundary layer in the vicinity of the plane. Furthermore, the above local Nusselt numbers along the curved surface are greater than those of the isothermal hemisphere at different Grashof numbers. Correspondently, the average Nusselt number increases with the increasing of the Grashof numbers, which is smaller than that of the isothermal hemisphere. Finally, the correlating equation of the average Nusselt number with the Grashof number at different thermal conditions for the hemisphere has been obtained, which provide a valid prediction for the case of natural convection heat transfer from the hemisphere to dispose of applications in several fields of engineering, particularly those electrical cooling.

1. Introduction

Natural convection is a type of heat transport, in which the fluid motion is not generated by any external source (like a pump, fan, suction device, etc.) but only by density differences occurring in the fluid due to the temperature gradients. It is prevalently encountered in industrial applications and our daily lives because of its simple formation mechanism described above. The spherical geometry is relevant to many industrial processes, such as vaporization and condensation of fuel droplets, manufacturing systems, the cooling for the spherical bodies in packed beds, and the heat transfer in many electronic components that are nearly spherical [1]. The hemispherical shape is applied to the thermal treatment of foodstuffs [2], microelectronics, semiconductor processing-related applications, the melting of polymer pellets, the fuel-injection systems in aviation applications, and the novel design of heat storage devices [3].

The natural convection heat transport of a sphere have been studied by relevant researches, hence it is necessary to properly introduce the previous research work here. For early investigation works of the sphere's natural convective heat transport, the case of high Grashof numbers based on the hypothesis of the boundary-layer theory was studied [4–8]; by contrary, some researchers explored the case of very small Grashof numbers [9–11]. Different investigators used experimental methods to study the problem. Those works presented the average Nusselt numbers in a broad range of Grashof numbers [12–15]. Some works have studied some interesting situations, such as the melting of a solid benzene sphere in benzene liquid [16], isothermal spheres in water [14], isothermal spheres in polymer solutions [17], and an ice sphere melting in water [18]. With regard to the analyses with using the numerical approach, a number of studies have been conducted. Geoola and Cornish [19] were the first ones to use the steady Navier-Stokes equations to simulate the natural convection heat transfer from a sphere in air. Subsequently, the local and average heat transfer rate for the Rayleigh number ranging from 10^{-1} to 10^4, have been reported by Farouk [20] via solving the Navier–Stokes equations. Recently, a numerically transient investigation for the heat transfer from a sphere cooled by natural convection, has been presented for $10^5 \le Gr \le 10^9$ and $Pr = 0.02$ by jia et at [21]. In the following research, they studied the steady-state natural convection over an isothermal sphere and obtained heat transfer rate and drag coefficients in a wide range of Grashof number and Prandtl numbers of 0.72 and 7.0 [22]. George Gogos et al. [22] presented the transient results for the range of Grashof numbers $10^5 \le Gr \le 10^9$ and three Prandtl numbers, namely, Pr = 0.02, 0.7, 7. Considering other thermal conditions, they studied the

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Nomenclature

C_D	total drag coefficient, dimensionless
C_f	frictional component of drag coefficient, dimensionless
C_P	pressure component of drag coefficient, dimensionless
g	gravitational acceleration, m/s^2
k	thermal conductivity, W/m·K
h	convective heat transfer coefficient, $W/m^2{\cdot}K$
$\overline{Nu}$	average Nusselt number, dimensionless
Nu_l	local Nusselt number, dimensionless
p	gauge pressure, Pa
p_∞	ambient fluid pressure, Pa
Pr	Prandtl number, dimensionless
Gr	Grashof number, dimensionless
D	diameter of the hemisphere, m
D_∞	width of computational domain, m
R	radius of the hemisphere
R^*	dimensionless radius of the hemisphere ($=R/D$)
S	surface area of the hemisphere, m^2
T	temperature of fluid, K
T_m	film temperature of fluid, K
T_∞	temperature of ambient fluid, K
T_w	temperature of the surface of the hemisphere, K
ΔT	temperature difference, ($=T_w - T_\infty$), K
r	dimensionless unit normal vector on the surface of hemisphere
u, v	x and y components of the velocity, m/s
U, V	x and y components of the velocity, dimensionless
V_θ	tangential velocity, m/s
V_R	radial velocity, m/s
V_θ^*	dimensionless tangential velocity ($=V_\theta/u_{ref}$)
V_r^*	dimensionless radial velocity ($=V_R/u_{ref}$)
u_{ref}	reference velocity, m/s
x, y	Cartesian coordinates, m)
X, Y	Cartesian coordinates (dimensionless)

Greek symbols

α	thermal diffusive coefficient, m2/s
β	coefficient of thermal expansion, K^{-1}
θ	hemisphere angle, degree
ϑ	kinematic viscosity, m^2/s
μ	dynamic viscosity, Pa·s
δ	distance between two grid points on the surface of hemisphere, m
τ	wall shear stress per unit length of hemisphere, Pa
ρ	density of the fluid, kg/m^3
ρ_∞	density of ambient fluid, kg/m^3

transient laminar natural convection heat transfer over a sphere subjected to a constant heat flux in the same range of the Grashof numbers and Prandtl numbers. Some relevant equations and analytical methods for a sphere are reviewed in those literature [23–25]. In industrial applications, we are more concern with the intensity of natural convective heat transfer, i.e., the average Nusselt number, and some correlating equations of the Nusselt number for a sphere are briefly reviewed in Table 1. Due to the difference of structure and thermal condition between the sphere and hemisphere, these empirical relationship equations are not applicable to describe the situation of the hemisphere. The above researches are about the natural convection of the sphere. However, this paper mainly studies the natural convection of the hemisphere. Consequently, it is vital to review literature on free convection from a hemisphere.

Snoek et al. [26] studied the natural convection of the hemisphere in a relatively narrow range of Rayleigh numbers ($5.36 \times 10^5 \le Ra \le 6.39 \times 10^5$) based on the radius of the hemispheres, and they obtained the overall rate of heat transfer in air. An experimental study of the natural convection flows over heated hemispheres with its curved surface oriented in the upward and downward directions in water ($Pr = 7$) at two surfaces conditions, namely, constant temperature and constant heat flux, in the range of Rayleigh numbers ($2.8 \times 10^8 \le Ra \le 2 \times 10^9$) based on diameter has been carried out by Jaluria and Gebhart [27]. They showed the detailed measurements of the velocity and temperature fields close to the hemisphere, compared the measured plume with the axisymmetric plume which would rise

Table 1
The studies of the intensity of natural convective heat transfer (all based on diameter).

Investigators	Equations	Conditions
Yuge [13]	$\overline{Nu} = 2 + 0.428{\cdot}Ra^{0.25}$	Heat transfer-air
Kyte.MaddenandPiret [43]	$\overline{Nu} = 2 + 0.339{\cdot}Ra^{0.25}$	Heat transfer-air
Ranz and Marshall [44]	$\overline{Nu} = 2 + 0.6{\cdot}\mathrm{Pr}^{0.0833}{\cdot}Ra^{0.25}$	Evaporation-drops
Kranse and Schenk [16]	$\overline{Nu} = 2 + 0.59{\cdot}Ra^{0.25}$	Melting-benzene
Garner and Keey [45]	$\overline{Nu} = 2 + 0.585{\cdot}Ra^{0.25}$	Mass transfer
Garner and Hoffman [46]	$\overline{Nu} = 5.4 + 0.44{\cdot}Ra^{0.25}$	Mass transfer
Van der Burgh [47]	$\overline{Nu} = 0.525{\cdot}Ra^{0.25}$	Melting-benzene
Schenkand Schenkels [18]	$\overline{Nu} = 0.56{\cdot}Ra^{0.25}$	Melting-ice spheres
akob and Linke [48]	$\overline{Nu} = 0.555{\cdot}Ra^{0.25}$	Heat transfer-various body shapes
Vanier [14]	$\overline{Nu} = 2 + 0.52{\cdot}Ra^{0.25}$	Melting-ice spheres
Boberg and Starrett [49]	$\overline{Nu} = 0.51{\cdot}Ra^{0.25}$	Heat transfer-transient method
Schiitz [50]	$\overline{Nu} = 2 + 0.59{\cdot}Ra^{0.25}$	Mass transfer-electrochemical
Waynes. Amato and Chi Tien [14]	$\overline{Nu} = 2 + (0.5 \pm 0.09){\cdot}Ra^{0.25}$	Heat transfer-water
Kanichi Saito et al. [51]	$\overline{Nu} = 2 + 0.531{\cdot}\mathrm{Pr}^{0.2}{\cdot}Ra^{0.25}$	Heat transfer- constant heat flux
Mathers et al. [12]	$\begin{cases}\overline{Nu} = 2 + 0.282{\cdot}Ra^{0.25}, \ Ra \le 10^2 \\ \overline{Nu} = 2 + 0.5{\cdot}Ra^{0.25}, \ 10^2 \le Ra \le 10^6\end{cases}$	Heat transfer-air
K. JAFARPUR [25]	$\overline{Nu} = 2 + \frac{0.589 \cdot Ra^{0.25}}{\left[1+\left(\frac{0.43}{\mathrm{Pr}}\right)^{0.5625}\right]^{0.444}}$	Heat transfer- isothermal
Churchill and Thelen [52]	$\overline{Nu} = 2 + \frac{Ra/5}{\left[1+\left(\frac{0.5}{\mathrm{Pr}}\right)^{0.5625}\right]^{1.78}}$	Heat transfer