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# Least square spectral collocation method for nonlinear heat transfer in moving porous plate with convective and radiative boundary conditions

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ABSTRACT

In this article, the least square spectral collocation method (LSSCM) is proposed to predict temperature distribution and heat transfer efficiency of moving porous plate. In this moving porous plate, heat is dissipated to ambient flow by radiation and convection, and generated by non-linear internal heat source. Two types of boundary conditions of this moving porous plate, constant temperature boundary condition, and combined convective and radiative boundary conditions are taken into account. Otherwise, radiation heat transfer in moving porous plate is also considered and assumed by Rosseland approximation. Different with traditional porous heat transfer, heat transfer equation of moving porous plate can be considered as a special kind of convective-diffusive equation with strong convection features. The convection term may cause non-physical oscillation of solutions. To overcome this non-physical oscillation, the least square scheme is adopted. Lagrange interpolation polynomials and Chebyshev collocation points are employed for spectral discretization. In order to validate LSSCM for this nonlinear heat transfer process, a test case is examined. The computational results by LSSCM agree well with analytical solutions, which shows that the present model is high accuracy and good flexibility to simulate nonlinear heat transfer in moving porous plate. Then, effects of thermo-physical parameters on dimensionless temperature and heat transfer efficiency are comprehensively investigated.

## 1. Introduction

During the past years, heat transfer through a heated porous medium has been a subject of considerable research and has evoked wide interest of many scholars due to its widely engineering applications [1]. These applications include but not limited to heat exchanger in high heat flux, combustion engine, heat pipe, pack-sphere bed, electronic cooling [2]. As early in 1985, Bejan and Khair [3] numerically investigated the phenomenon of natural convection heat and mass transfer in a porous medium. Kiwan and Al-Nimr [4] introduced the concept of using porous medium to enhance heat transfer from a given surface. Compared with solid surface, porous medium increases heat exchange area and decreases weight by removing the solid material with hollow pores. Kiwan and Zeitoun [5] utilized the Darcy model to describe the solid-fluid interactions in the porous medium and proposed the finite volume model to estimate the performance of porous extended surface attached around the inner layer of the annular space between two concentric cylindrical enclosures. They further found that the cylindrical porous medium enhanced convective heat transfer coefficient over 70% compared with that of traditional solid surface under natural convective environment. Bhanja et al. [6] established an analytical model to obtain temperature, efficiency and optimum design parameters of moving porous medium which loses heat to ambient flow by convection and radiation. Das [7] adopted forward and inverse method to analyze coupled conductive, convective and radiative heat transfer in porous fin with cylindrical profile.

In the traditional thermal analysis of porous medium, thermo-physical properties are assumed to be constant for reducing the mathematical complexity of the energy equation. However, under some actual conditions, these assumptions maybe invalid. Convective heat transfer coefficient is the power exponent function of the temperature difference between extended surface and ambient fluid, namely, $h \propto (\Delta T)^m$, where the value of power index $m$ depends on the mechanism of convective heat transfer. Sertkaya et al. [8] experimentally investigated $m = 1/4$ for laminar natural convective heat transfer and $m = 1/3$ for turbulent natural convective heat transfer. Fu et al. [9] experimentally validated that the total hemispherical emissivity varied with temperature. Heat generation is also dependent on temperature [10,11]. Shateri and Salahsour [12] using least square method to predict temperature distribution and heat performance of longitudinal porous fins with various

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**Nomenclature**

| | |
|---|---|
| $A$ | Cross-section area of moving porous plate, m$^2$ |
| $A_{i,j}$ | spectral coefficient matrix defined in Eq. (30) |
| $B_i$ | spectral coefficient matrix defined in Eq. (31) |
| $C_1$, $C_2$ | constants of analytical solution |
| $c_1$, $c_2$, $c_3$ | heat generation parameters |
| $c_p$ | specific heat capacity, J·kg$^{-1}$·K$^{-1}$ |
| $D_{i,j}^{(1)}$ | the element of the first order derivative matrix |
| $D_{i,j}^{(2)}$ | the element of the second order derivative matrix |
| Da | Darcy number |
| $F_{i,j}$ | spectral coefficient matrix defined in Eq. (33) |
| $g$ | gravitational acceleration, m·s$^{-2}$ |
| $G_i$ | spectral coefficient matrix defined in Eq. (34) |
| Gr | Grash of number |
| $h$ | convective heat transfer coefficient, W·m$^{-2}$·K$^{-1}$ |
| $h_i$ | Lagrange interpolation polynomials |
| $k$ | thermal conductivity, W·m$^{-1}$·K$^{-1}$ |
| $k_{\text{eff}}$ | efficient thermal conductivity, W·m$^{-1}$·K$^{-1}$ |
| $k_r$ | thermal conductivity ratio |
| $K$ | permeability of the porous plate, m$^2$ |
| $L$ | length of the plate, m |
| $m$ | power index of convective heat transfer coefficient |
| $\dot{m}$ | mass flow rate passing through porous medium, kg·s$^{-1}$ |
| $N$ | number of collocation points |
| $N_{\text{cc}}$ | convective-conductive parameter |
| $N_{\text{rc}}$ | radiative-conductive parameter |
| $p$ | perimeter of moving porous plate, m |
| Pe | Peclet number |
| Pr | Prandtl number |
| $q$ | heat transfer rate, W |
| $q_{\text{con}}$ | conductive heat transfer rate, W |
| $q_{\text{rad}}$ | radiative heat transfer rate, W |
| $\dot{q}$ | volumetric heat generation rate, W·m$^{-3}$ |
| $Q_a$ | dimensionless heat generation rate at ambient temperature |
| $R$ | Rosseland parameter |
| $R(x)$ | residual error |
| $R_{\text{Benchmark}}$ | benchmark solution |
| $R_{\text{LSSCM}}$ | numerical solution obtained by the LSSCM |
| Ra | Rayleigh number |
| $s_i$ | Chebyshev-Gauss-Lobatto collocation points |
| $S$ | source term |
| $S_h$ | porous parameter |
| $T$ | temperature, K |
| $\tilde{u}$ | trail function |
| $v$ | speed of the moving porous plate, m·s$^{-1}$ |
| $v_w$ | velocity of fluid passing through the porous plate, m·s$^{-1}$ |
| $w_i$ | coefficient of Lagrange interpolation polynomials |
| $W_i$ | weight function |
| $x$ | coordinate in $x$-direction, m |
| $X$ | dimensionless axial coordinate |

*Greek Symbols*

| | |
|---|---|
| $\alpha$ | coefficient of surface emissivity |
| $\beta$ | coefficient of thermal expansion, K$^{-1}$ |
| $\beta_R$ | Rosseland extinction coefficient, m$^{-1}$ |
| $\delta$ | thickness of the porous plate, m |
| $\varepsilon$ | surface emissivity |
| $\varepsilon_{\text{error}}$ | integral averaged relative error |
| $\eta$ | heat transfer efficiency |
| $\Theta$ | dimensionless temperature |
| $\upsilon_f$ | Kinematic viscosity of the fluid, m$^2$·s$^{-1}$ |
| $\rho$ | density, Kg·m$^{-3}$ |
| $\sigma$ | Stefan-Boltzmann constant, W·m$^{-2}$·K$^{-4}$ |
| $\varphi$ | porosity of the porous medium |
| $\psi$ | dimensionless plate length |
| $\Im$ | the differential operator |

*Subscripts*

| | |
|---|---|
| $a$ | values at ambient fluid |
| $b$ | values at base |
| $f$ | values of fluid |
| $i, j$ | solution node indexes |
| $L$ | values at tip |
| $s$ | values of solid material |
| $x$ | values at $x$ |
| $x + dx$ | values at $x + dx$ |

*Superscripts*

| | |
|---|---|
| * | The latest iterative value |

profiles. In these porous fins, heat transfer coefficient, surface emissivity and heat generation rate vary with temperature. Ma et al. [13] considered the case of porous fin with temperature dependent convective heat transfer coefficient, surface emissivity and heat generation. In this paper, spectral collocation method (SCM) was used to assess the thermal performance of porous fin. Hatami and Ganji [14] investigated nonlinear heat transfer through porous fin with different porous materials and temperature dependent heat generation.

In the case of medium with high temperature, radiation heat transfer from medium plays an important role in combined heat transfer process. Then, the effect of radiation heat transfer should be included in evaluating the temperature distribution of medium. For example, Golar and Bakier [15] studied the natural convection and radiation in the porous fin. They found that the radiation transferred more heat than a similar model without radiation. Sheikholeslami et al. [16–20] investigated the effect of thermal radiation on combined heat transfer in ferrofluid flow.

In recent years, considerable attention has been devoted to the study of heat transfer in moving porous medium. Bhanja et al. [6] found that porous medium in moving condition can provide the higher heat transfer rate than that in stationary condition. Moradi et al. [21] used homotopy analysis method to simulate combined natural convection and radiation in moving porous fin. Homotopy analysis method [22–24] is one of nonlinear analytical techniques, and has been successfully applied to lots of different types of nonlinear problems in science and engineering. Recently, Ma et al. [25] developed spectral element method to solve coupled conductive, convective and radiative heat transfer in moving porous fins with irregular profiles. However, radiative heat transfer within porous medium and combined convective and radiative boundary conditions were not considered.

In this paper, thermal radiation heat transfer in porous medium and combined convective and radiative boundary conditions are both taken into account. Thus, the corresponding heat transfer equation can be considered as a special kind of convective-diffusive equation with strong convection features. In the process of numerical simulation, the convection term may cause non-physical oscillation of solution [26]. This type of instability can occur in many numerical methods, such as finite different method, finite volume method, finite element method and element differential method [27]. Special stabilization techniques such as upwind scheme or artificial viscosity are used in finite volume method and finite element method [28]. Least square method (LSM) is one of stabilization techniques for highly nonlinear equation. Atouei