International Journal of Thermal Sciences 132 (2018) 344–354

Contents lists available at ScienceDirect

International Journal of Thermal Sciences

journal homepage: www.elsevier.com/locate/ijts

Simultaneous effects of melting heat and internal heat generation in stagnation point flow of Jeffrey fluid towards a nonlinear stretching surface with variable thickness

Tasawar Hayat[a,b], Rai Sajjad Saif[c], Rahmat Ellahi[c], Taseer Muhammad[d,*], Ahmed Alsaedi[b]

[a] Department of Mathematics, Quaid-I-Azam University, Islamabad, 44000, Pakistan
[b] Nonlinear Analysis and Applied Mathematics (NAAM) Research Group, Department of Mathematics, Faculty of Science, King Abdulaziz University, Jeddah, 21589, Saudi Arabia
[c] Department of Mathematics and Statistics, Faculty of Basic & Applied Sciences, International Islamic University, Islamabad, 44000, Pakistan
[d] Department of Mathematics, Government College Women University, Sialkot, 51310, Pakistan

ARTICLE INFO

Keywords:
Stagnation point flow
Jeffrey fluid
Melting heat process
Internal heat generation
Nonlinear stretching surface
Variable surface thickness

ABSTRACT

This paper investigates the magnetohydrodynamic (MHD) stagnation point flow of Jeffrey material towards a nonlinear stretching surface with variable surface thickness. Heat transfer characteristics are examined through the melting process, viscous dissipation and internal heat generation. A nonuniform applied magnetic field is considered. Boundary-layer and low magnetic Reynolds number approximations are employed in the problem formulation. Both the momentum and energy equations are converted into the non-linear ordinary differential system using appropriate transformations. Convergent solutions for resulting problems are computed. Behaviors of various pertinent parameters on velocity and temperature distributions are studied in detail. Further the heat transfer rate is also computed and analyzed.

1. Introduction

The study of non-Newtonian fluid has gained considerable attention because of its widespread applications in chemical and petroleum industries, biological sciences and geophysics. In particular, boundary layer flow of non-Newtonian fluid over a stretching surface are prevalent in several industrial processes, for example, drawing of plastic films, extrusion of a polymer sheet from a dye, oil recovery, food processing, paper production and numerous others. The well-known Navier-Stokes relations are inadequate to describe the flow behavior of non-Newtonian materials. However various constitutive relations of non-Newtonian materials are proposed in the literature due to their versatile nature. The under discussion fluid model is known as Jeffrey fluid model [1–5]. This model describes the characteristics of ratio of relaxation to retardation times and retardation time.

It is noted that the stretching of sheet is not necessarily always linear for example in plastic production process. Thus some researchers discussed the problem of non-linear stretching surface for different cases of fluid flow. Gupta and Gupta [6] was the first who proposed that the stretching of surface may not necessarily be linear. Later on, flow and heat transfer over a non-linear stretching sheet is studied by Vajravelu [7]. Cortell [8] extended the work of [7] by considering prescribed surface temperature and constant surface temperature conditions. Prasad et al. [9] investigated mixed convection flow over a nonlinear stretching surface with variable fluid characteristics. Yazdi et al. [10] studied the slip flow over a nonlinear permeable stretched surface with chemical reaction. Mustafa et al. [11] discussed the axisymmetric flow of nanofluid over a nonlinear stretching surface. Recently Hayat et al. [12] examined the magnetohydrodynamic flow of second grade nanomaterial by a nonlinear stretched surface.

The analysis of boundary-layer flow over a stretching surface with variable thickness has many technological applications. However much attention has not been paid to this concept of variable thickness of surface. Few studies have been presented in this direction. Fang et al. [13] examined the boundary layer flow of viscous fluid over a stretched sheet with variable thickness. Khader and Megahed [14] studied the boundary layer flow of viscous fluid over a nonlinear stretching surface with variable thickness and slip velocity. Subhashini et al. [15] considered the stretching sheet with variable thickness and find the dual solutions for thermal diffusive flow. Abdel-Wahed et al. [16] considered the nanofluid flow of viscous fluid over a moving sheet with variable thickness. Recently Hayat et al. [17] investigated the homogenous-heterogenous reactions in nanofluid flow over a nonlinear stretched surface with variable thickness. On the other hand the process of heat

* Corresponding author.
E-mail addresses: taseer_qau@yahoo.com, taseer@math.qau.edu.pk (T. Muhammad).

https://doi.org/10.1016/j.ijthermalsci.2018.05.047
Received 5 November 2016; Received in revised form 29 May 2018; Accepted 30 May 2018
1290-0729/

Nomenclature

u, v	velocity components $[m.\ s^{-1}]$	Nu_w	local Nusselt number
μ	dynamic viscosity $[Pa.\ s]$	x, y	coordinate axes $[m]$
τ	Cauchy stress tensor $[N.\ m^{-2}]$	ρ	density of fluid $[kg.\ m^{-3}]$
ν	kinematic viscosity $[m^2.\ s^{-1}]$	$\mathbf{b}$	body force $[N.\ m^{-3}]$
T	temperature $[K]$	U_e	free stream velocity $[m.\ s^{-1}]$
T_∞	ambient fluid temperature $[K]$	T_m	melting temperature $[K]$
σ	electrical conductivity $[S.\ m^{-1}]$	T_0	constant temperature $[K]$
α_1	thermal diffusivity $[m^2.\ s^{-1}]$	B_0	magnetic field strength $[N.\ m^{-1}.\ A^{-1}]$
λ_1	ratio of relaxation to retardation times	k	thermal conductivity $[W.\ m^{-1}.\ K^{-1}]$
c_p	specific heat $[J.\ K^{-1}]$	λ_2	retardation time $[s]$
U_w	surface stretching velocity $[m.\ s^{-1}]$	c_s	heat capacity of solid surface $[W.\ s.\ K^{-1}.\ kg^{-1}]$
λ	latent heat of fluid $[W.\ s.\ kg^{-1}]$	Q^*	heat generation coefficient
η	dimensionless variable	n	shape parameter
f'	dimensionless velocity	ε	small constant
A	ratio parameter	θ	dimensionless temperature
β	Deborah number	M	magnetic parameter
Ec	Eckert number	Pr	Prandtl number
Me	melting parameter	δ	heat generation parameter
		α	surface thickness parameter
		Re_x	local Reynolds number

transfer with melting analysis has received much interest in the field of silicon wafer process, magma solidification and melting of permafrost [18]. Thus Tien and Yen [19] performed an analysis to analyze the behavior between the fluid and melting body through forced convection heat transfer. Later on steady laminar flow over a flat surface with melting heat transfer is studied by Epstein and Cho [20]. Mixed convective heat transfer by a vertical plate in a liquid saturated porous medium with melting effect is analyzed by Cheng and Lin [21]. Yacob et al. [22] studied the micropolar fluid flow over stretching/shrinking surface with melting heat transfer. Few other studies in this direction can be seen through the attempts [23–27].

The objective of present communication is to analyze the combined effects of melting heat and internal heat generation in stagnation-point flow of Jeffrey material towards a nonlinear stretched surface. Stretching surface is of variable thickness. The process of heat transfer is examined in the presence of melting heat transfer, viscous dissipation and internal heat generation. A nonuniform magnetic field is taken into account. The considered flow analysis has relevance with the metallurgical process. Such flows have specific relevance to the cooling of continuous strips and filaments drawn through a quiescent fluid, the purification of molten metals from non-metallic inclusions etc. Further the control of flow of electrically conducting fluid by electromagnetic force in industry is recognized as a useful tool for handling materials like sodium metals and semi conducting melts. Homotopy analysis technique [28–35] is used to compute the serious solutions of velocity and temperature. Influences of various embedded flow parameters on the velocity and temperature are presented and discussed in detail. Moreover local Nusselt number is computed and examined.

2. Modeling

Here the steady two-dimensional (2D) stagnation point flow of Jeffrey fluid is considered. The flow is generated by a non-linear stretching surface. The $x-$ and $y-$axes are taken along and perpendicular to the stretching surface respectively. A non-uniform magnetic field of strength B_0 is applied in the $y-$direction (see Fig. 1). We assume

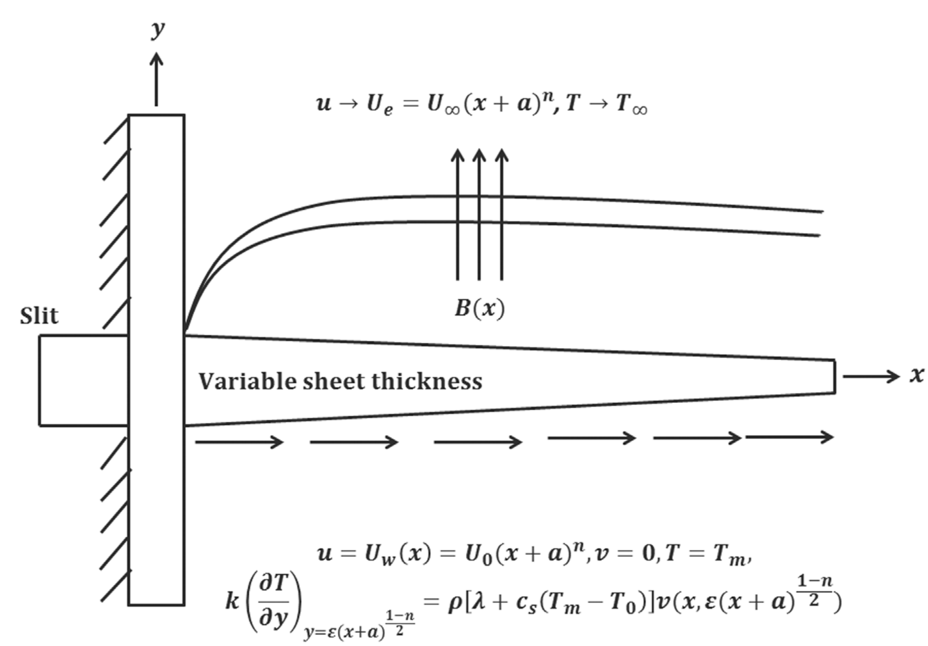

Fig. 1. Geometry of the problem.