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Cascaded lattice Boltzmann method for thermal flows on standard lattices

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ABSTRACT

In this paper, a thermal cascaded lattice Boltzmann method (TCLBM) is developed in combination with the double-distribution-function (DDF) approach on the standard D2Q9 lattice. A density distribution function relaxed by the cascaded scheme based on central moments is employed to solve the flow field, and a total energy distribution function relaxed by the BGK scheme is used to solve the temperature field. The two distribution functions are coupled naturally to provide a new TCLBM. In this method, the viscous heat dissipation and compression work are taken into account, the Prandtl number and specific-heat ratio are adjustable, and the external force is considered directly without the Boussinesq assumption. The TCLBM is validated by numerical experiments of the thermal Couette flow, low-Mach number shock tube problem, Rayleigh-Bénard convection, and natural convection in a square cavity with a large temperature difference. The simulation results agree well with the analytical solutions and/or results given by previous researchers.

1. Introduction

The lattice Boltzmann method (LBM), based on the kinetic theory, has achieved remarkable success as an alternative method to conventional computational fluid dynamics (CFD) for thermal flow and heat transfer applications during the past three decades [1–9]. Different from solving the discretized Navier-Stokes (N-S) equations in traditional CFD methods, the LBM solves a discrete kinetic equation at the mesoscopic scale, designed to reproduce the N-S equations in the macroscopic limit. The main advantages for LBM over traditional CFD include [10,11]: convenience to deal with complex boundary, easiness of programming, high parallel efficiency, and natural incorporation of micro and meso-scale physics.

The basic algorithm realization of LBM is collision-streaming or streaming-collision, although other time and space evolution schemes can also be used. To be specific, at each time step the collision is first locally executed and followed by streaming the post-collision distributions to their neighbors, or just exchanging the above procedure [12]. Based on this algorithm, various collision operators can be adopted, such as the single-relaxation-time (SRT) or BGK operator [13], two-relaxation-time (TRT) operator [14,15], multiple-relaxation-time (MRT) operator [16,17], and entropic operator [18–20]. Compared with these extensively used operators, cascaded or central moment operator, first proposed by Geier et al. [21] in 2006, is more recent. The collision in the cascaded Lattice Boltzmann method (CLBM) is performed by relaxing central moments to their local equilibrium values separately, which is different from MRT LBM where the raw moments are relaxed. As mentioned in Ref. [21], central moments can be expressed as polynomials of raw moments of the same order and below. When a raw moment is relaxed (in MRT), all central moments at the same or higher orders will be changed. This "cross-talk" is a source of instability and can be removed in CLBM. By choosing the relaxation parameters properly, CLBM can be adopted to simulate very high Reynolds number flows using coarse grids without adopting any turbulence models or entropic stability [21]. Recently, Lycett-Brown and Luo [22] extended the CLBM to multiphase flow using the interaction potential method [23] with the EDM force scheme [24]. Compared with the LBGK method, the proposed model provided significant improvement in reducing spurious velocities, and increasing the stability range for the Reynolds number and liquid to gas density ratio. They further extended the model to three dimensions and achieved high Weber number, high Reynolds number and high density ratio simultaneously in binary droplet collision simulations [25,26]. More recently, based on a generalized multiple-relaxation-time (GMRT) framework, we proposed a consistent method to incorporate a force field into CLBM and clarified the relation between CLBM and MRT LBM [27].

Although CLBM has obtained success in high Reynolds number single-phase flows and multiphase flows, its applications are so far

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limited to incompressible flows. Recently, we showed that for incompressible thermal flows, CLBM can improve the numerical stability significantly compared with the BGK model [9]. The purpose of the present study is to extend CLBM to Low-Mach compressible thermal flows. Generally, there are three feasible ways to construct thermal LBMs. The first one, multispeed approach [28,29], is a straightforward extension of athermal to thermal LBMs, in which more discrete velocities are adopted to match higher-order moment constraints of the density distribution function for recovering the energy equation. In the second one, a density distribution function is still used to simulate the velocity field, while other methods, such as finite difference or finite volume [30,31], are adopted for the temperature field. The double-distribution-function (DDF) [1,2] approach is the third one, where two different distribution functions are adopted to solve flow and temperature fields, respectively. In DDF-based thermal LBMs, the compression work and heat dissipation can be simply included, and the specific-heat ratio and Prandtl number are adjustable. On the whole, the DDF approach keeps the intrinsic features and simple structures of the standard LBM, and more comparisons and discussions among the three methods can be found in Refs. [2,4,32]. In the history, the first DDF thermal model was proposed by He et al. [1] by using an internal-energy-distribution-function-based DDF approach. Guo et al. [2] then presented another DDF thermal model using a total energy distribution function to solve the energy equation, which is simpler than He and coworkers' model. In Guo and co-authors' model, the local temperature in equilibrium density and energy distribution functions is replaced by the reference temperature, thus it is a decoupling model and is limited to Boussinesq flows. In 2012, Li et al. [4] developed a coupling DDF thermal model which can simulate more general thermal flows, and the model was extended to three-dimensions by Feng et al. [33] recently. Inspired by these works, we construct a thermal cascaded lattice Boltzmann method (TCLBM) in the present work based on the DDF approach. In the TCLBM, a density distribution function is relaxed using the cascaded scheme, a total energy distribution function is relaxed using the SRT scheme, and the external force is considered directly without the Boussinesq assumption.

The rest of the paper is structured as follows: Section 2 briefly introduces the cascaded LBM. Section 3 presents a method to incorporate the force field into cascaded LBM. In Section 4, we extend the athermal CLBM to TCLBM. Numerical experiments are carried out for several benchmark problems to validate the proposed model in Section 5. Finally, conclusions of this work are made in Section 6.

2. Cascaded LBM

In this paper, the D2Q9 lattice [13] is adopted, and the discrete velocities are defined as $\mathbf{e}_0 = (0,0)$, $\mathbf{e}_a = (\cos[(a-1)\pi/2], \sin[(a-1)\pi/2])c$, for a=1–4, and $\mathbf{e}_a = (\sqrt{2}\cos[(a-9/2)\pi/2], \sqrt{2}\sin[(a-9/2)\pi/2])c$ for a=5–8. In LBM, $c = \delta_x/\delta_t$, here δ_x and δ_t are the lattice spacing and time step, and $c = \delta_x = \delta_t = 1$ is used in this work. For the derivation of CLBM, we follow Lycett-Brown and Luo [22] and begin with the velocity moments of the discrete distribution function (DF) f_a, and then f_a and f_a^{eq} can be formulated as functions of the corresponding moments and equilibrium moments.

The raw moments are defined as

$$\rho M_{mn} = \sum_a f_a e_{ax}^m e_{ay}^n. \tag{1}$$

in this notation, the zero-order moment $M_{00} = 1$, and first-order moments $M_{10} = u_x$ and $M_{01} = u_y$ are conserved, corresponding to mass, x and y momentum components, respectively. To get the formulations of f_a , another six independent moments are needed, including 3 second-order moments (M_{11}, M_{02} and M_{20}), two third-order moments (M_{21}, M_{12}, noting that M_{03} and M_{30} are not independent of the first-order ones owing to the lack of symmetry in D2Q9 lattice), and the fourth-order moment M_{22}. Recombining the second-order moments, the trace of the pressure tensor, the normal stress difference and the off diagonal element of the pressure tensor are given by

$$E = M_{20} + M_{02}, \quad N = M_{20} - M_{02}, \quad \Pi = M_{11}. \tag{2}$$

According to the definition above, we get the raw moment representation of populations:

$$f_0 = \rho[M_{00} - E + M_{22}], \tag{3a}$$

$$f_1 = \frac{1}{2}\rho\left[M_{10} + \frac{1}{2}(E+N) - M_{12} - M_{22}\right], \tag{3b}$$

$$f_2 = \frac{1}{2}\rho\left[M_{01} + \frac{1}{2}(E-N) - M_{21} - M_{22}\right], \tag{3c}$$

$$f_3 = \frac{1}{2}\rho\left[-M_{10} + \frac{1}{2}(E+N) + M_{12} - M_{22}\right], \tag{3d}$$

$$f_4 = \frac{1}{2}\rho\left[-M_{01} + \frac{1}{2}(E-N) + M_{21} - M_{22}\right], \tag{3e}$$

$$f_5 = \frac{1}{4}\rho[\Pi + M_{21} + M_{12} + M_{22}], \tag{3f}$$

$$f_6 = \frac{1}{4}\rho[-\Pi + M_{21} - M_{12} + M_{22}], \tag{3g}$$

$$f_7 = \frac{1}{4}\rho[\Pi - M_{21} - M_{12} + M_{22}], \tag{3h}$$

$$f_8 = \frac{1}{4}\rho[-\Pi - M_{21} + M_{12} + M_{22}]. \tag{3i}$$

It should be noted that other variables can also be expressed using their moments in this form similarly.

Central moments are defined in a reference frame shifted by the local velocity,

$$\rho\widetilde{M}_{mn} = \sum_a f_a (e_{ax} - u_x)^m (e_{ay} - u_y)^n. \tag{4}$$

The transformation between the raw moments and central moments can be expressed using the binomial theorem as given by Lycett-Brown and Luo [22]. To construct a CLBM, we follow the assumption adopted in Ref. [34], by setting the discrete equilibrium central moments equal to the corresponding continuous values,

$$\rho\widetilde{M}_{mn}^{eq} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f^{eq}(\xi_x - u_x)^m(\xi_y - u_y)^n d\xi_x\xi_y, \tag{5}$$

where f^{eq} is the local Maxwell-Boltzmann distribution for athermal fluid at temperature T_0 in continuous particle velocity space (ξ_x, ξ_y),

$$f^{eq} = \frac{\rho}{2\pi RT_0}\exp\left[-\frac{(\xi - \mathbf{u})^2}{2RT_0}\right], \tag{6}$$

and the lattice sound speed $c_s = \sqrt{RT_0}$ is set to be $1/\sqrt{3}$ in this work. Substituting Eq. (6) into Eq. (5), we can calculate the second order and above central moments, and write them using the combination as done in raw moments:

$$\widetilde{\Pi}^{eq} = \widetilde{N}^{eq} = \widetilde{M}_{21}^{eq} = \widetilde{M}_{12}^{eq} = 0, \qquad \widetilde{E}^{eq} = 2RT_0, \quad \widetilde{M}_{22}^{eq} = (RT_0)^2. \tag{7}$$

The implementation of CLBM is also composed of collision step and streaming step. For the collision step, central moments are relaxed to their equilibrium values, separately:

$$\widetilde{\Pi}^* = w_1\widetilde{\Pi}^{eq} + (1 - w_1)\widetilde{\Pi}, \tag{8a}$$

$$\widetilde{N}^* = w_1\widetilde{N}^{eq} + (1 - w_1)\widetilde{N}, \tag{8b}$$

$$\widetilde{E}^* = w_2\widetilde{E}^{eq} + (1 - w_2)\widetilde{E}, \tag{8c}$$