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Thermal radiation therapy of biomagnetic fluid flow in the presence of localized magnetic field

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ABSTRACT

Present work describes the applications of magnetic field and thermal radiation on the two-dimensional, unsteady flow of bio-magnetic fluid (blood) in a rectangular vessel. The viscosity of the fluid is considered to be an exponential function of temperature. The radiative heat flux is approximated with the help of Stefan Boltzmann law, which is a nonlinear function of temperature. The governing nonlinear, coupled system of partial differential equations for the underlying problem is represented in terms of stream function-vorticity formulation, which is further solved numerically with the help of upwind scheme along with successive over relaxation method. For the validation of the solutions, results are compared with the numerical and experiemental data available in the literature. In order to notice the influence of localized magnetic field and thermal radiation, solutions are presented graphically in terms of streamlines, isotherms, vorticity function contours and velocity component contours and are discussed both qualitatively and quantitatively from the physiological point of view.

1. Introduction

Biomagnetic fluid dynamics (BFD) is a fascinating area in fluid mechanics due to the fact that it has attractive applications in bio-engineering and medical science. Particularly, its importance can be seen in the development of magnetic devices for cell separation, solving problems of cardiovascular and respiratory systems, magnetic wound treatment or cancer tumor treatment causing magnetic hyperthermia, targeted transport of drugs using magnetic particles as drug carriers, reduction of bleeding during surgeries or provocation of occlusion of the feeding vessels of cancer tumors and development of magnetic tracers (see Refs. [1–9]). Besides this, deep heat muscular treatment has also been discussed by Refs. [10,11] in the long tube of circular section. Ogulu and Bestman [10] revealed mathematically about the reason behind a hot bath that physicians recommend for cuts and also the reason why physiotherapists use ice packs for bruises.

In BFD, it has been observed that the principles of ferrohydrodynamics (FHD) and magnetohydrodynamics (MHD) are served as a foundation for the blood flow. Considering this, Haik et al. [12] initially established the formulation of BFD model by assuming blood as Newtonian, homogeneous, and electrically non-conducting fluid. Tzirtzilakis [13] further proposed the extended model of BFD, which also encounters the electrical conductivity effects. The problem of biomagnetic fluid flow in a channel was studied by Loukopoulos and Tzirtzilakis [14] in which they assumed that magnetization linearly varies with temperature as well as magnetic field intensity. Tzirtzilakis and Loukopoulos [15] also investigated the effect of strong localized magnetic field in a channel filled will biofluid. They concluded that the electrical conductivity of blood should not be ignored near the area where the magnetic field is uniformly distributed. Papadopoulos and Tzirtzilakis [16] established some numerical results for the flow in curved square duct and noticed that magnetic field may be used for controlling the blood flow by magnetic means. The fundamental problem of the biomagnetic fluid flow in a lid driven cavity under the influence of a steady localized magnetic field was examined by Tzirtzilakis and Xenos [17].

The models considered above deals with the constant viscosity of fluid in the vessel/cavity and reported the influence of magnetic field on the characteristics of the flow. But, generally, the viscosity of bio fluids undergoes considerable variations with respect to the temperature. For instance, the viscosity of water reduces to 240% when the temperature is increased from 10°C to 50°C. This trend is not only

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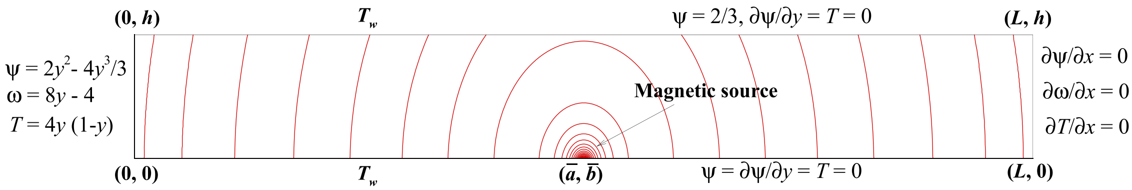

Fig. 1. Geometry of the vessel/channel.

noticed in low viscosity fluids like water, but can also happens in other highly viscous fluids like glycerin, blood etc. Quantitative data reveals that the blood viscosity increases 26.13% by reducing the temperature from 36.5°C to 22°C, which results in reduction in the blood flow rate by 20.72% (see Ref. [18]). Therefore, it can be deduced that temperature dependent viscosity extensively changes the solution of heat and fluid flow as compared to those fluids which have constant viscosity. In this regard, influence of variable viscosity of blood flow in the channel was discussed by Siddiqa et al. [19,20] in different physical conditions and reported interesting results.

In pathological situations, thermal radiation therapy is one of the treatments employed by medical practitioners [21–23], which describes the procedure that involve heat transmission, below the skin surface, into muscles and tissues. Electromagnetic heat, such as shortwaves and microwaves, sends the heat up to 2 inches into the tissue and muscles. It works best for injuries in joints, muscles, and tendons. Moreover, hyperthermia has been demonstrated as an effective treatment during cancer therapy in recent years. Its objective is to raise the temperature of pathological tissues above cytotoxic temperature 41°C to 45°C without over exciting the healthy tissues. The understanding of the behavior of thermal radiation therapy along with localized magnetic field is important in order to determine the relevance of thermal dose in clinical treatments. In this regard, theoretical studies dealing with the combined effects of thermal radiation and localized magnetic field on the transient flow of blood in a large artery with viscous dissipation has not been discussed in the literature. Therefore, in this study, we will present the numerical solutions of simultaneous effects of thermal radiation and localized magnetic field. The viscosity of the fluid (blood) is considered to be temperature dependent and it varies exponentially as proposed by Torrance and Turcotte [24]. The model considered here is based on the principles of bio-magnetic fluid dynamics. The stream function-vorticity formulation is employed on coupled system of nonlinear partial differential equations and the resulting system is then solved by adopting successive over relaxation (SOR) technique. The numerical results are presented in terms of streamlines, temperature contours, velocity contours and vorticity contours for the important physical parameters. In section 2 mathematical model is given, which is later nondimensionalized and transformed into stream function-vorticity formulation. In section 3 algorithm is explained briefly, which is used to develop FORTRAN code for the simulation of the problem. In section 4 results are discussed for different parameters, which emerge in section 2. Lastly, in section 5 conclusions are made based on the behavior of important parameters that occurs during the formulation of the problem.

2. Mathematical model

Flow of incompressible, unsteady, two-dimensional, electrically conducting, Newtonian bio-magnetic (blood) fluid passing through the rectangular vessel is considered here. It is assumed that the length and width of the vessel are L and h respectively. The flow is subject to a magnetic source (placed near lower plate) and thermal radiation. At the inlet, the flow is considered to be fully developed, for which the distribution of velocity is adopted as proposed by Ref. [26]. The plates and fluid temperature are respectively set to be T_w and T_f, provided $T_w < T_f$. The schematic of the problem is described in Fig. 1 where the rectangular coordinates are chosen accordingly. In the figure, the origin of the coordinates is located at the leading edge of the vessel. The rotational forces acting on the erythrocytes when entering the magnetic field are neglected. The upper and lower walls of the channel are supposed to be electrically non-conducting and the electric field is ignored. The viscosity of the fluid is considered to be a strong function of temperature as proposed by Torrance and Turcotte [24]. Under the above conditions, the equations that govern the flow can be stated as:

$$\frac{\partial\bar{u}}{\partial\bar{x}} + \frac{\partial\bar{v}}{\partial\bar{y}} = 0 \tag{1}$$

$$\rho\left(\frac{\partial\bar{u}}{\partial\bar{t}} + \bar{u}\frac{\partial\bar{u}}{\partial\bar{x}} + \bar{v}\frac{\partial\bar{u}}{\partial\bar{y}}\right) = -\frac{\partial\bar{p}}{\partial\bar{x}} + \nabla.\,(\bar{\mu}\nabla\bar{u}) + \mu_0\bar{M}\frac{\partial\bar{H}}{\partial\bar{x}} - \sigma\bar{B}^2\bar{u} \tag{2}$$

$$\rho\left(\frac{\partial\bar{v}}{\partial\bar{t}} + \bar{u}\frac{\partial\bar{v}}{\partial\bar{x}} + \bar{v}\frac{\partial\bar{v}}{\partial\bar{y}}\right) = -\frac{\partial\bar{p}}{\partial\bar{y}} + \nabla.\,(\bar{\mu}\nabla\bar{v}) + \mu_0\bar{M}\frac{\partial\bar{H}}{\partial\bar{y}} \tag{3}$$

$$\rho c_p\left(\frac{\partial\bar{T}}{\partial\bar{t}} + \bar{u}\frac{\partial\bar{T}}{\partial\bar{x}} + \bar{v}\frac{\partial\bar{T}}{\partial\bar{y}}\right) + \mu_0\bar{T}\frac{\partial\bar{M}}{\partial\bar{T}}\left(\bar{u}\frac{\partial\bar{H}}{\partial\bar{x}} + \bar{v}\frac{\partial\bar{H}}{\partial\bar{y}}\right) - \sigma\bar{B}^2\bar{u}^2 = \kappa\nabla^2\bar{T} + \bar{\mu}\bar{\phi} - \bar{\nabla}\cdot\bar{q_r} \tag{4}$$

where

$$\bar{\phi} = 2\left(\frac{\partial\bar{u}}{\partial\bar{x}}\right)^2 + 2\left(\frac{\partial\bar{v}}{\partial\bar{y}}\right)^2 + \left(\frac{\partial\bar{u}}{\partial\bar{x}} + \frac{\partial\bar{v}}{\partial\bar{y}}\right)^2, \quad \bar{q_r} = -\frac{4\sigma_0}{3(a_r + \sigma_s)}\bar{\nabla}\bar{T}^4,$$
$$\bar{\mu} = \mu_i\exp\left[\lambda\left(\frac{1}{2} - \frac{(\bar{T} - T_w)}{(T_f - T_w)}\right)\right]$$

are the mathematical expressions for the dissipation function, radiative heat flux and variable viscosity, respectively. In the above system of equations, $(\bar{u}, \bar{v})$ are the velocity components in the $(\bar{x}, \bar{y})$ directions, $\bar{T}$ is the temperature of the fluid, $\bar{p}$ the pressure, ρ the magnetic fluid density, σ the electric conductivity, $\bar{B}$ the magnetic induction defined by $\bar{B} = \mu_0\bar{H}$, μ_0 the magnetic vacuum permeability, c_p the specific heat at constant pressure, $\bar{M}$ the magnetization, $\bar{H}$ the magnetic field strength, κ the thermal conductivity, μ the variable viscosity, μ_i the reference viscosity, λ the variable viscosity parameter, σ_0 the Stefan Boltzman constant, a_r the Rosseland mean absorption coefficient and σ_s the scattering coefficient.

The boundary conditions are:

$$\begin{aligned}
&\text{Inlet} && (\bar{x} = 0, 0 \le \bar{y} \le h): && \bar{u} = \bar{u}(\bar{y}),\ \bar{v} = 0,\ \bar{T} = \bar{T}(\bar{y})\\
&\text{Outlet} && (\bar{x} = L,\ 0 \le \bar{y} \le h): && \frac{\partial\bar{u}}{\partial\bar{x}} = \frac{\partial\bar{v}}{\partial\bar{x}} = \frac{\partial\bar{T}}{\partial\bar{x}} = 0\\
&\text{Upper plate} && (\bar{y} = h,\ 0 \le \bar{x} \le L): && \bar{u} = \bar{v} = 0,\ \bar{T} = T_w\\
&\text{Lower plate} && (\bar{y} = 0, 0 \le \bar{x} \le L): && \bar{u} = \bar{v} = 0,\ \bar{T} = T_w
\end{aligned} \tag{5}$$

$\bar{u}(\bar{y})$ and $\bar{T}(\bar{y})$ represents the parabolic profiles for the velocity and temperature distributions respectively. In this work we have considered the empirical relationship introduced by Ref. [25] for magnetization $\bar{M}$, which varies with magnetic field intensity $\bar{H}$ and temperature $\bar{T}$, i.e.,

$$\bar{M} = K\bar{H}(T_c - \bar{T}) \tag{6}$$

where K is the constant and T_c is the Curie temperature. The