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ORIGINAL PAPER

Area 4

The finite-time ruin probability of a risk model with stochastic return and Brownian perturbation

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Abstract

This paper investigates a renewal risk model with stochastic return and Brownian perturbation, where the price process of the investment portfolio is described as a geometric Lévy process. When the claim sizes have a subexponential distribution, we derive the asymptotics for the finite-time ruin probability of the above risk model. The obtained result confirms that the asymptotics for the finite-time ruin probability of the risk model with heavy-tailed claim sizes are insensitive to the Brownian perturbation.

Keywords Asymptotics · Finite-time ruin probability · Brownian perturbation · Lévy process · The class of subexponential distributions

Mathematics Subject Classification 62P05 · 62E10 · 91B30

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1 Introduction

Consider a renewal risk model with stochastic return and Brownian perturbation. In such a model, the successive claim sizes $X_n, n \geq 1$, form a sequence of independent nonnegative random variables (r.v.s) with common distribution F, and the inter-arrival times $\theta_n, n \geq 1$, form another sequence of independent and identically distributed (i.i.d.) nonnegative r.v.s, which are independent of $\{X_n, n \geq 1\}$. The claim arrival times $\tau_n = \sum_{k=1}^{n} \theta_k, n \geq 1$, constitute a renewal counting process

$$N(t) = \sup\{n \geq 0 : \tau_n \leq t\}, \quad t \geq 0,$$

with a finite mean function $\lambda(t) = EN(t), t \geq 0$. The insurer is allowed to make risk-free and risky investments. The price process of the investment portfolio is described as a geometric Lévy process $\{e^{R_t}, t \geq 0\}$, here $\{R_t, t \geq 0\}$ is a Lévy process, which starts from zero and has independent and stationary increments. The discounted aggregate claims up to time $t \geq 0$ is denoted by

$$D(t) = \sum_{i=1}^{N(t)} X_i e^{-R_{\tau_i}}. \tag{1.1}$$

Then the discounted value of the surplus process with stochastic return on investment can be express by

$$U(t) = x + \int_0^t c(s) e^{-R_s} ds - D(t) + \delta \int_0^t e^{-rs} dB(s), \quad t \geq 0, \tag{1.2}$$

where $x \geq 0$ is the initial risk reserve of an insurance company, $c(t) \geq 0$ is the density function of premium income at time t, $\delta \geq 0$ is the volatility factor, $r \geq 0$ is the force of interest and $B(t), t \geq 0$ is the diffusion perturbation which is a standard Brownian motion. Throughout the paper, we assume that the premium density function $c(t)$ is bounded, i.e. $0 \leq c(t) \leq c_0$ for some $c_0 > 0$ and for all $t \geq 0$. As usual, we assume that $\{X_n, n \geq 1\}$, $\{\theta_n, n \geq 1\}$, $\{R_t, t \geq 0\}$ and $\{B(t), t \geq 0\}$ are mutually independent. For any $t \geq 0$, the finite-time ruin probability of the above risk model is defined as

$$\psi(x, t) = P\left(\inf_{0 \leq s \leq t} U(s) < 0 \middle| U(0) = x\right).$$

For the above risk model without Brownian perturbation (i.e. $\delta = 0$), when $R_t = rt$ for all $t \geq 0$, there are some earlier literatures to deal with the ruin probabilities for the independent or dependent claim sizes (see, e.g. [1,4,10,12–14,23,25,29,30]). When $\{R_t, t \geq 0\}$ is a Lévy process, Tang et al. [27] investigated the ruin probabilities of this risk model with regularly varying claim sizes, in which the claim sizes and the inter-arrival times are two sequences of independent r.v.s. Li [15] considered a time-dependent risk model with extended-regularly-varying claim sizes, in which there existed a certain dependence between the claim sizes and the inter-arrival times. Yang