International Journal of Thermal Sciences 132 (2018) 486–497

Contents lists available at ScienceDirect

International Journal of Thermal Sciences

journal homepage: www.elsevier.com/locate/ijts

Detection of contact failures with the Markov chain Monte Carlo method by using integral transformed measurements

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ABSTRACT

This work deals with the solution of an inverse heat conduction problem aiming at the detection of contact failures in layered composites through the estimation of the contact conductance between the layers. The spatially varying contact conductance is estimated using a Bayesian formulation of the problem and a Markov chain Monte Carlo method, with infrared camera measurements of the transient temperature field on the surface of the body. The inverse analysis is formulated using a data compression scheme, where the temperature measurements are integral transformed with respect to the spatial variable. The present approach is evaluated using synthetic measurements and experimental data from controlled laboratory experiments. It is shown that only few transformed modes of the data are required for solving the inverse problem, thus providing substantial reduction of the computational time in the Markov chain Monte Carlo method, as well as regularization of the ill-posed problem.

1. Introduction

The detection of internal failures in materials is a subject of extensive research due to its importance in several fields, for example, in structural health monitoring [1–14]. With the recent advancement and practical applications (e.g., in the aeronautic, space and petroleum industries) of composites consisting of layers of different materials, nondestructive and noninvasive methods for the detection of adhesion failures between the composite layers have been developed [2–15]. Heat transfer techniques can be found among these methods, by using qualitative [1,2,7,13,14] as well quantitative analyses based on the solutions of inverse problems [3–12].

In our previous works [3,4], the contact failures were detected through the estimation of the contact conductance between the layers of different materials from measurements of the transient temperature over the surface of the composite. The approach used in Refs. [3,4] was based on a Bayesian formulation of the inverse problem, with a total variation prior model for the unknowns and using the Markov chain Monte Carlo (MCMC) method for the inference of the Bayesian model. The computational complexity of MCMC with large dimensional problems is often prohibitive. Therefore, in this paper, we extend [3,4] to accommodate a data compression scheme. The temperatures measured with an infrared camera are spatially compressed through the integral transformation with eigenfunctions related to the actual physical problem. Only a few transformed modes are then used in the inverse analysis and the forward model is formulated directly in terms of the transformed (compressed) temperatures. A similar data compression approach was used in Refs. [16,17] for the estimation of spatially varying properties in a one-dimensional problem and is applied here with a two-dimensional transformation. The data compression applied in this work not only reduces the computational time required for the Markov chain Monte Carlo method, but also provides regularization for the inverse problem [18]. Conceptually, the integral transform data compression scheme falls within the broader class of orthogonal decomposition methods, such as POD – Proper Orthogonal Decomposition, Principal Component Analysis, Karhunen–Loeve decomposition and Truncated Singular Value Decomposition [19–25]. The accuracy of the proposed methodology is examined with simulated measurements, as well as with actual thermographic data obtained with controlled laboratory experiments [9], involving samples manufactured with

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https://doi.org/10.1016/j.ijthermalsci.2018.06.006
Received 2 July 2017; Received in revised form 27 March 2018; Accepted 4 June 2018

Nomenclature

a,b,c	dimensions of the plate
$Bi_c(X,Y)$	dimensionless contact conductance
D	total number of measurements
$h_c(x,y)$	thermal contact conductance
k	thermal conductivity
k_{max}	number of transient measurements
$q(x,y,t)$	heat flux imposed on the top boundary
M	number of elements in the spatial grid
$\mathbf{P}$	vector of parameters
T	temperature
T_o	initial temperature in the medium
X,Y,Z	dimensionless spatial coordinates
Z_1	dimensionless position of the contact interface
$\mid \mathbf{A} \mid$	determinant of matrix ($\mathbf{A}$)

Greeks

α	thermal diffusivity
β	eigenvalue given by equation (6.a)
ϕ	eigenfunction given by equation (4.a)
γ	eigenvalue given by equation (6.b)
φ	eigenfunction given by equation (4.b)
θ	dimensionless estimated temperature
$\mathbf{\Theta}$	vector of estimated dimensionless temperatures
τ	dimensionless time
ψ	dimensionless measured temperature
$\mathbf{\Psi}$	vector of measured temperatures

Subscripts

1,2	plates 1 and 2, respectively
i,j	order of the eigenquantities in the X and Y directions, respectively
ref	reference values

Superscripts

*	dimensionless thermophysical properties and heat flux
$\_$	transform in the X direction
$\sim$	transform in the Y direction

designed contact failures of different formats.

2. Physical problem and mathematical formulation

The physical problem considered here involves heat conduction through a plate with two layers, heated through its top surface by a heat flux q (x,y,t), as illustrated by Fig. 1. The bottom surface of the plate is thermally insulated and heat transfer is assumed negligible through its lateral surfaces. The plate is initially at a uniform temperature, T_o, and the physical properties of each layer are assumed homogeneous and not dependent on temperature. The length and width of the plate are a and b, respectively, while its thickness is denoted by c. A spatially distributed contact resistance between the two adjacent layers is modeled by a contact conductance h_c (x,y) [26]. For the inverse analysis, measurements of the temperature at the top (heated) surface of the plate, obtained with an infrared camera, are available.

The mathematical problem is written in dimensionless form by using the following variables:

$$\theta(X,Y,Z,\tau)=\frac{T(x,y,z,t)-T_o}{T_o}\qquad q^*(X,Y,\tau)=q(x,y,t)\frac{c}{k_{ref}T_o}\tag{1.a,b}$$

$$k^*=\frac{k}{k_{ref}}\qquad \alpha^*=\frac{\alpha}{\alpha_{ref}}\tag{1.c,d}$$

$$X=\frac{x}{c}\quad Y=\frac{y}{c}\quad Z=\frac{z}{c}\quad Z_1=\frac{c_1}{c}\quad A=\frac{a}{c}\quad B=\frac{b}{c}\tag{1.e-j}$$

$$\tau=\frac{\alpha_{ref}t}{c^2}\qquad Bi_c(X,Y)=\frac{h_c(x,y)c}{k_{ref}}\tag{1.k,l}$$

and we obtain:

$$\partial_\tau\theta_1(X,Y,\tau)=\alpha_1^*\nabla^2\theta_1 \text{ in } 0<X<A,\ 0<Y<B,\ 0<Z<Z_1,\ \text{for } \tau>0\tag{2.a}$$

$$\partial_\tau\theta_2(X,Y,\tau)=\alpha_2^*\nabla^2\theta_2 \text{ in } 0<X<A,\ 0<Y<B,\ Z_1<Z<1,\ \text{for } \tau>0\tag{2.b}$$

$$\partial_Z\theta_1=0 \text{ at } Z=0 \text{ in } 0<X<A,\ 0<Y<B \text{ and } \tau>0\tag{2.c}$$

$$k_1^*\partial_Z\theta_1=k_2^*\partial_Z\theta_2 \text{ at } Z=Z_1 \text{ in } 0<X<A,\ 0<Y<B \text{ and } \tau>0\tag{2.d}$$

$$k_1^*\partial_Z\theta_1=Bi_c(X,Y)[\theta_2-\theta_1] \text{ at } Z=Z_1 \text{ in } 0<X<A,\ 0<Y<B \text{ and } \tau>0\tag{2.e}$$

$$k_2^*\partial_Z\theta_2=q^*(X,Y,\tau) \text{ at } Z=1 \text{ in } 0<X<A,\ 0<Y<B \text{ and } \tau>0\tag{2.f}$$

$$\partial_X\theta_1=\partial_X\theta_2=0 \text{ at X}=0,\ 0<Y<B,\ 0<Z<1, \text{ and } \tau>0\tag{2.g}$$

$$\partial_X\theta_1=\partial_X\theta_2=0 \text{ at } X=A, 0<Y<B, 0<Z<1, \text{ and } \tau>0\tag{2.h}$$

$$\partial_Y\theta_1=\partial_Y\theta_2=0 \text{ at } Y=0, 0<X<A, 0<Z<1, \text{ and } \tau>0\tag{2.i}$$

$$\partial_Y\theta_1=\partial_Y\theta_2=0 \text{ at } Y=B, 0<X<A, 0<Z<1, \text{ and } \tau>0\tag{2.j}$$

$$\theta_1=\theta_2=0 \text{ at } \tau=0 \text{ in } 0 \text{ in } 0<X<A,\ 0<Y<B,\ 0<Z<1\tag{2.k}$$

where the contact interface is located at $Z=Z_1$.

3. Forward problem

The forward (direct) problem associated with the formulation given by equation (2.a-k) involves the determination of the temperature fields $\theta_1(X,Y,Z,\tau)$ and $\theta_2(X,Y,Z,\tau)$. The direct problem corresponding to the transformed data is solved here by using a hybrid analytical-

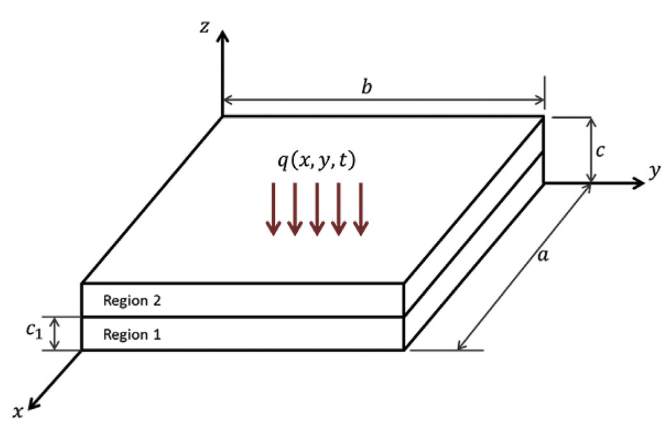

Fig. 1. Physical problem.