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Non-Fourier heat conduction/convection in moving medium

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ABSTRACT

Non-Fourier one-dimensional unsteady heat conduction in a moving medium is investigated by using the Cattaneo-Vernotte- Christov-Jordan (CVCJ) heat flux model for medium speeds less than (sub-critical), equal to (critical), and greater than (super-critical) the thermal wave speed. Coupled partial differential equations are solved simultaneously by a finite volume numerical method. Temperature and heat flux distributions for sub-critical, critical, and super-critical flow conditions are presented for two example problems. The importance of boundary conditions on the thermal wave propagation in both sub-critical and super-critical cases is discussed. Approximate analytical solutions are presented which qualitatively substantiate the numerical results.

1. Introduction

Non-Fourier heat transfer in a stationary medium has been studied extensively by many researchers using various analytical and numerical methods [1–4]. In these studies, the Cattaneo-Vernotte (CV) heat flux equation is used [5,6]. In one space dimension the CV model is described by the equations

$$q + \tau\frac{\partial q}{\partial t} = -\kappa\frac{\partial T}{\partial x} \tag{1a}$$

$$c = \sqrt{\frac{\kappa}{\rho c_p \tau}} \tag{1b}$$

In Eq. (1a) q is the heat flux, T is the temperature, x is position, t is time, κ is the thermal conductivity, and τ is the thermal relaxation time. In Eq. (1b) ρ is the density, c_p is the specific heat, and c is the finite thermal wave speed. For Fourier conduction the relaxation time τ is zero and the thermal wave speed becomes infinite. In many small scale and rapid transient heat transfer problems however, finite thermal wave speeds are observed. They are very small in metals (~pico-s) but there are materials for which these values are much larger, such as amorphous materials (~10 s) and biological tissues (~100 s) [7].

Christov and Jordan [8,9] pointed out that in a moving medium, the CV model should be modified to make the constitutive behavior independent of rigid rotations. For rigid translation of the medium with speed u it is sufficient to replace the time derivative appearing in Eq. (1a) by the co-moving derivative to get

$$q + \tau\left(\frac{\partial q}{\partial t} + u\frac{\partial q}{\partial x}\right) = -\kappa\frac{\partial T}{\partial x} \tag{2}$$

It is shown in Ref. [8] that Eq. (1a) does not predict the correct wave behavior in a moving medium, but Eq. (2) does. The present paper focuses attention on Cattaneo-Vernotte-Christov-Jordan (CVCJ) heat flux model, Eq. (2), applied to one-dimensional unsteady heat transfer in a medium moving with a constant speed.

Non-Fourier heat transfer in a moving medium has been recently studied by many investigators [10–18]. The CVCJ heat transfer model has been used in several non-Fourier applications involving moving media [10–18]. Only [10–12] [14], and [18]) involve either a rigid medium translating with a constant speed or a heat source translating with a constant speed. Of these, the most pertinent to the present work are the papers by Gomez et al. [11], Al-Khairy and Al-Ofey [12], and Al-Khairy [14]. In Ref. [11] the exact solution for steady one-dimensional CVCJ heat transfer is presented for both sub-critical ($u< c$) and super-critical ($u> c$) conditions. It is shown that a finite element scheme equivalent to central finite differencing fails to reproduce this exact solution in the vicinity of critical ($u= c$) conditions. This is similar to the high Peclet number problem in the numerical solution of convection/diffusion problems. In Ref. [12] a Laplace transform solution is reported for one-dimensional unsteady CVCJ heat transfer due to a distributed heat source in a semi-infinite constant speed medium with a vanishing temperature gradient at the boundary. This solution is extended to a finite constant speed medium with vanishing temperature gradients at both boundaries in Ref. [14]. Only sub-critical and critical conditions are considered in these papers.

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In Refs. [11] [12], and [14] the energy balance equation and the heat flux constitutive equation are combined to produce a single partial differential equation for the temperature. This will be called the conventional formulation in the present discussion. Since the resulting equation is second order in both time and space, two initial and two boundary conditions for temperature are necessary and sufficient to create a properly posed problem for the temperature. Once this problem has been solved the heat flux constitutive equation is then solved separately using the known temperature. In the present work the finite volume method is used to obtain numerical solutions to the equations governing the CVCJ model. It was found that application of the finite volume approach to the conventional formulation appeared to work well for sub-critical and critical cases but failed in super-critical cases. To overcome this problem associated with super-critical applications, a formulation has been used which retains a heat flux term in the partial differential equation for the energy. This equation and heat flux constitutive equation are then solved simultaneously by the finite volume method in sequence for the temperature and heat flux distributions. As discussed below, this approach appeared to work well over the entire range of medium speeds. Heat transfer in a moving medium with constant speed is the simplest example of convection/conduction heat transfer. Thus, the present work is both of interest in itself and for extension to more general non-Fourier conduction/convection heat transfer problems.

The remainder of this document is organized as follows. In Section 2 the two formulations of the governing equations for one-dimensional unsteady CVCJ heat transfer in a moving medium discussed above are presented. Numerical models based on a finite volume method for both formulations are given in Section 3. Applications of these approaches for two example problems are reported in Section 4. In Section 5, some approximate analytical solutions are presented to support qualitatively the numerical results obtained by the proposed numerical model. Section 6 summarizes the work and important conclusions.

2. Governing equations

The enthalpy form of the energy balance equation may be written as

$$\frac{\partial}{\partial t}(\rho h) + \frac{\partial}{\partial x}(\rho u h) = -\frac{\partial q}{\partial x} + \dot{Q} \tag{3}$$

In Eq. (3) h is the specific enthalpy, $h = c_p T$, and $\dot{Q}$ is the rate of energy generation per unit volume. Even though only homogeneous media are considered in the present study, the enthalpy form of the energy equation is employed to facilitate future extension to heterogeneous media with phase change. Differentiating Eq. (2) with respect to x and assuming a constant medium velocity and relaxation time produces

$$\tau\left[\frac{\partial}{\partial t}\left(\frac{\partial q}{\partial x}\right) + u\frac{\partial}{\partial x}\left(\frac{\partial q}{\partial x}\right)\right] + \frac{\partial q}{\partial x} = -\kappa\frac{\partial^2 T}{\partial x^2} \tag{4}$$

In the conventional approach all $\frac{\partial q}{\partial x}$ terms appearing in Eq. (4) are eliminated by using Eq. (3). Solving for $\frac{\partial q}{\partial x}$ from Eq. (3), substituting the result into Eq. (4), and using Eq. (1b) leads to

$$\frac{\partial}{\partial t}(\rho h) + \frac{\partial}{\partial x}\left[\rho u h - \frac{\kappa}{c_p}\left(1 - \frac{u^2}{c^2}\right)\frac{\partial h}{\partial x}\right] = \dot{Q} - \tau\frac{\partial^2}{\partial t^2}(\rho h) + \tau\frac{\partial \dot{Q}}{\partial t} - 2\tau u\frac{\partial^2}{\partial x \partial t}(\rho h) + \tau u\frac{\partial \dot{Q}}{\partial x} \tag{5}$$

For $u > c$ it can be seen that the effective thermal diffusion coefficient $\frac{\kappa}{c_p}\left(1 - \frac{u^2}{c^2}\right)$ appearing in Eq. (5) becomes negative. Eq. (5) can be solved with two initial and two boundary conditions for the temperature (enthalpy). Using this approach in conjunction with the finite volume numerical method was found to succeed for $u \le c$ but fail for $u > c$. It is believed that this failure is due to the negative effective thermal conductivity mentioned above.

In the conventional formulation Eq. (5) is decoupled from Eq. (2) and in most available references heat flux results are not reported. An alternate version of the heat flux equation can be obtained by substituting $\frac{\partial q}{\partial x}$ obtained from Eq. (3) into Eq. (2) to yield

$$\tau\frac{\partial q}{\partial t} + q = -\frac{\partial}{\partial x}\left[\frac{\kappa}{c_p}\left(1 - \frac{u^2}{c^2}\right)h\right] + \tau u \rho\frac{\partial h}{\partial t} - \tau u\dot{Q} \tag{6}$$

Eq. (6) can be solved explicitly for the new time level heat flux with the known new time level enthalpy (temperature) distribution obtained by Eq. (5) at each time step.

In the present formulation, the second $\frac{\partial q}{\partial x}$ term in the bracket in Eq. (4) is retained. This produces

$$\frac{\partial}{\partial t}(\rho h) + \frac{\partial}{\partial x}\left(\rho u h - \frac{\kappa}{c_p}\frac{\partial h}{\partial x}\right) = \dot{Q} - \tau\frac{\partial^2}{\partial t^2}(\rho h) + \tau\frac{\partial \dot{Q}}{\partial t} - \tau u\frac{\partial^2}{\partial x \partial t}(\rho h) + \tau u\frac{\partial^2 q}{\partial x^2} \tag{7}$$

There are two major differences between Eq. (5) and Eq. (7). First, the effective thermal diffusion coefficient $\frac{\kappa}{c_p}$ appearing in Eq. (7) is always positive. Second, Eq. (7) contains the heat flux term and must be solved simultaneously with Eq. (2) for the temperature and heat flux distributions. It was found that the use of this formulation in conjunction with the finite volume numerical method overcame the difficulties discussed earlier associated with the conventional formulation for super-critical cases.

3. Numerical method

Detailed discussions of the finite volume method adopted for the present study are contained in Refs. [19,20]. A brief description of the finite volume approximation of Eq. (7) is presented below. The finite volume approximation of Eq. (5) (not presented explicitly herein) is almost identical.

Eq. (7) is integrated over a nominal control volume as shown in Fig. 1a. The resulting finite volume representation is

$$a_p h_p = a_E h_E + a_W h_W + b \tag{8}$$

In Eq. (8)

$$a_E = D_e A(|p_e|) + \max(-F_{em}, 0) \tag{8a}$$

$$a_W = D_w A(|p_w|) + \max(F_{wm}, 0) \tag{8b}$$

The diffusion conductance and local Peclet number at the interface of control volumes are identical to those of Fourier conduction/convection problems, namely

$$D_{e,w} = \frac{\left(\frac{\kappa}{c_p}\right)_{e,w}}{(\delta x)_{e,w}}\ ;\ P_{e,w} = \frac{F_{e,w}}{D_{e,w}};\ F_{e,w} = (\rho u)_{e,w} \tag{8c}$$

Using the power law scheme for the function $A(|p_{e,w}|)$ yields

$$A(|p_{e,w}|) = \mathrm{abs}[0, (1 - 0.1P_{e,w})^5] \tag{8d}$$

The modified flux due to a finite relaxation time is

$$F_{e,w,m} = \left(1 + \frac{\tau}{\Delta t}\right)F_{e,w} \tag{8e}$$

The coefficient of h_p is

$$a_P = a_E + a_W + a_P^0\left(1 + \frac{\tau}{\Delta t}\right) \tag{8f}$$

In Eq. (8f)

$$a_P^0 = \frac{\rho_p \Delta x_p}{\Delta t} \tag{8g}$$

The source term in Eq. (8) is