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Evidence-theory-based model validation method for heat transfer system with epistemic uncertainty

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ABSTRACT

In numerical heat transfer, the model validation problem with respect to epistemic uncertainty, where only a small amount of experimental information is available, has been recognized as a challenging issue. To overcome the drawback of traditional probabilistic methods in dealing with limited data, this paper proposes a novel model validation approach by using evidence theory. First, the evidence variables are adopted to characterize the uncertain input parameters, where the focal elements are expressed as mutually connected intervals with basic probability assignment (BPA). In the subsequent process of predicting response focal elements, an interval collocation analysis method with small computational cost is presented. By combining the response BPAs in both experimental measurements and numerical predictions, a new parameter calibration method is then developed to further improve the accuracy of computational model. Meanwhile, an evidence-theory-based model validation metric is defined to test the model credibility. Eventually, the famous Sandia thermal challenge problem is utilized to verify the feasibility of presented model validation method in engineering application.

1. Introduction

In thermal engineering, the experimental tests and computational simulations are the two important means for system analysis. The large number of experimental tests can obtain the intuitive and reliable results, but the expenses are always considerable, especially for complex systems [1,2]. With rapid development of modern computer technology, the computational simulations play an increasingly important role in engineering due to the relatively small cost. However, the strong dependency on computational models creates a critical issue of quantifying credibility in simulation accuracy, which can provide the decision-maker with the necessary information [3–5]. Model validation, defined as *the process of determining the degree to which a model is an accurate representation of the real world from the perspective of the intended uses of the model* [6], is just the general technology for characterizing this credibility of computational model before practical application. In recent years, the model validation problem has received considerable attentions and intensive investigations from many professional societies and national laboratories [7–9]. The American Institute of Aeronautics and Astronautics (AIAA) and the American Society of Mechanical Engineers (ASME) published guidance documents for model validation in computational fluid dynamics and computational solid mechanics, respectively [10,11]. The U.S. Department of Energy (DoE) emphasized the importance of model validation in the Accelerated Strategic Computing Initiative program [12].

As is known to all, uncertainties are widely involved in the real world due to the unpredictable environment factors, inevitable measurement errors and incomplete knowledge [13–15]. Thus, compared with the traditional model validation activities in deterministic framework, the uncertainty-based model validation is more feasible and practical [16,17]. Generally speaking, the uncertainties can be classified into two categories: aleatory uncertainty and epistemic uncertainty [18]. Using the sufficient sample statistical information, the aleatory uncertainty is usually quantified as random variable or stochastic process by probability theory. Up to now, a lot of investigations have been conducted on aleatory uncertainty-based model validation [19–22]. Based on stochastic uncertainty propagation and data transformations, Chen et al. proposed a generic model validation method, where the number of required physical tests can be efficiently reduced [23]. In order to characterize the coherence between predictions and observations in random uncertain circumstance, four kinds of validation metrics, namely classical hypothesis testing, Bayes factor, frequentist's metric and area metric, were verified by a group of mathematical examples [24]. Using Bayesian updates and prediction related

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rejection criteria, Babuska et al. developed a systematic probabilistic approach for model validation [25]. Considering the high stochastic dimension of modeling uncertainties, a nonparametric probabilistic approach was investigated to perform the prior model in validating process [26]. Besides, for the three famous challenge problems proposed by Sandia National Laboratories, a lot of research results have been obtained in the probabilistic framework [27–30].

In contrast to the aleatory uncertainty analysis with sufficient available information, the epistemic uncertainty is more challenging because of the incomplete knowledge, especially in the case of limited data [31,32]. Several quantification methods, such as convex model [33], fuzzy set [34], interval variable [35] and evidence theory [36], have been proposed to characterize epistemic uncertainty, where the evidence theory is considered to be the most capable. It is because the concepts in evidence theory, such as focal element, basic probability assignment and so on, can be flexibly defined and utilized, which means the evidence theory could provide equivalent transformations to other models by necessary extensions. Along with the widespread concern in the recent two decades, evidence theory has obtained many achievements in uncertainty quantification and reliability analysis [37–40]. To describe the imprecise data, Bae et al. developed a novel epistemic uncertainty quantification technique by evidence theory, which can be considered as an effective alternative to the classical probabilistic methods [41]. Based on the Jacobi polynomial, Yin et al. proposed an evidence-theory-based method for response analysis of acoustic system [42]. To improve the computational efficiency of epistemic uncertainty analysis, Xie et al. presented a radial point interpolation method in evidence theory [43]. In the research work of Helton and Oberkampf, the performance of evidence theory in reliability analysis was summarized by a simple algebraic function [44]. Using the experiment design technique, Zhang et al. presented an efficient response surface method to evaluate the structural reliability in evidence theory [45]. From the overall perspective, evidence theory shows excellent superiority in epistemic uncertainty characterization and response prediction. Unfortunately, its application in model validation has not been reported by now.

In this study, a novel model validation method based on evidence theory is proposed for the engineering heat transfer system under epistemic uncertainties, which can efficiently assess the credibility of computational model. The structure of this paper is organized as follows. The fundamental concepts in evidence theory are firstly reviewed in Section 2. Subsequently, by using evidence variables to quantify the input uncertainties, an interval collocation analysis method is proposed in Section 3 to efficiently predict the response focal elements. In order to improve the prediction accuracy of computational model, a parameter calibration framework is established in Section 4 by updating response BPA. The famous Sandia thermal challenge problem is provided as the numerical example in Section 5 to verify the performance of proposed method. Finally, we conclude the paper with a brief discussion in Section 6.

2. Fundamental concepts in evidence theory

Evidence theory, also named as the Dempster-Shafer (DS) theory, was proposed by Dempster and Shafer to characterize the epistemic uncertainty [36]. As the basis of evidence theory, some fundamental concepts, such as frame of discernment (FD), basic probability assignment (BPA), combination rule and so on, are firstly reviewed in this section.

The frame of discernment (FD) Θ is defined as an exhaustive set, which is consisted of a group of mutually exclusive propositions as follows

$$\Theta = \{\theta_1, \theta_2, ..., \theta_n\} \tag{1}$$

where θ_i denotes the elementary proposition; n is the number of elementary propositions.

Subsequently, all subsets of FD Θ can construct a power set 2^Θ, which is adopted to represent all possible various propositions

$$2^\Theta = \{\Phi, \{\theta_1\}, \{\theta_2\}, ..., \{\theta_n\}, \{\theta_1, \theta_2\}, \{\theta_1, \theta_3\}, ..., \{\theta_1, \theta_n\}, ..., \Theta\} \tag{2}$$

where Φ stands for the empty set, and the total number of elements in 2^Θ is 2^n.

In evidence theory, the probability can be assigned to any element in the power set 2^Θ. In other words, not only the elementary proposition but also the proposition combinations can respectively obtain the independent probability, which is able to reasonably represent the imprecise probability information. This kind of probability description is called as the basic probability assignment (BPA), which can be denoted by a function m: $2^\Theta \to [0,1]$ mapping from 2^Θ to [0,1].

Similar with the probability density function in probability theory, the BPA is used to quantify the elementary belief measure of each proposition in evidence theory. Thus, for any proposition $A \in 2^\Theta$, three kinds of BPA conditions should be satisfied

$$\begin{aligned} &(i) \quad m(A) \geq 0 \ \ for \ \ any \ \ A \in 2^\Theta \\ &(ii) \quad m(\Phi) = 0 \\ &(iii) \quad \textstyle\sum_{A\in 2^\Theta} m(A) = 1 \end{aligned} \tag{3}$$

where the proposition A with positive BPA $m(A) > 0$ is named as the focal element.

In many cases, the evidential information may come from different sources. Thus, it is crucial to combine the available information to update the BPA. For the same FD Θ, assume that two independent BPAs m_1 and m_2 have been derived from two different sources. Introducing B and C to respectively express the corresponding propositions, then the popular Dempster-based combination rule can be formulated as follows to update the BPA m

$$m(A) = \begin{cases} 0 & if \ \ A = \Phi \\ \frac{\sum_{B\cap C=A} m_1(B)\times m_2(C)}{1-K} & if \ \ A \neq \Phi \end{cases} \tag{4}$$

where

$$K = \sum_{B\cap C=\Phi} m_1(B) \times m_2(C) \tag{5}$$

stands for all the inconsistent information.

Another combination rule, named as Yager-based combination rule, is considered to be more suitable to the problem with strongly inconsistent information, where the updated BPA is calculated by

$$m(A) = \begin{cases} 0 & if \ \ A = \Phi \\ \sum_{B\cap C=A} m_1(B) \times m_2(C) & if \ \ A \neq \Phi \ \ and \ \ \Theta \\ \sum_{B\cap C=A} m_1(B) \times m_2(C) + K & if \ \ A = \Theta \end{cases} \tag{6}$$

where K is the same as that in Eq. (5).

3. Response prediction with input evidence variables

In evidence theory, the elementary propositions in FD as shown in Eq. (1) can exist in various forms. However, in practical heat transfer system, the parameter epistemic uncertainty usually exhibits a certain fluctuation around its nominal value. Thus, it is reasonable to adopt continuous intervals to denote the elementary propositions in FD.

3.1. Input parameter characterization by evidence variables

First, the system input parameters with epistemic uncertainty can be modeled as l independent evidence variables as follows

$$\mathbf{X} = (X_1, X_2, ..., X_l) \tag{7}$$

where $X_i \in [\underline{X}_i, \overline{X}_i]$ stands for the evidence variable; the interval $[\underline{X}_i, \overline{X}_i]$ represents the FD range of X_i; $\mathbf{X}$ is the evidence variable vector.

For each evidence variable X_i, the elementary propositions in FD