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An inverse problem to assess the two-component unsteady wall shear rate

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ABSTRACT

The instantaneous two-dimensional wall shear rate is assessed through an inverse problem using mass transfer data from a three-segment electrodiffusion probe. The method is validated numerically in complex flow conditions involving (i) high amplitude periodic fluctuations on both wall shear rate magnitude and direction and (ii) direct numerical simulation (DNS) data from a turbulent three-dimensional channel flow. The approach is shown to outperform every other post-treatments available for mass transfer sensors, especially regarding its versatility and application range. The impact of the three-segment probe gap size is also examined numerically.

1. Introduction

Every wall confined flow is subject to wall shear stress, affecting the efficiency of many industrial systems such as pumps, turbines, heat exchangers or any application implying fluid circulation. Despite the considerable efforts in developing new methods over the years, measurement of wall shear stress remains a challenge, especially when both time and space resolution are required. Among the many available methods, the floating element is interesting due to the probe size and considerably large bandwidth [sometimes up to 4 kHz, see 1]. However, spatial resolution of the floating element is generally limited by the overall electronic components surrounding the probe and those sensors can rarely measure at the same time the shear stress direction. The hot-film anemometer has also been widely used to assess wall shear rate in unsteady flows. On paper, hot-film frequency response stays flat up to a few kHz [2], but this value is largely weakened considering heat conduction through the wall, which also introduces a bias error. Regardless of the many adaptations developed over the last few decades (among other things: reduction of the substrate thermal conductivity, creation of a vacuum cavity below the sensor), this problem persists and accurate measurements under unsteady conditions are still an issue [3]. Natural convection caused by the probe heating, known as the induced buoyancy, can also locally alter the flow conditions. He et al. [4] attributed the large dispersions in their calibration data to this phenomenon, observing discrepancies as high as ± 10 % (turbulent pipe flow with). Although flow direction can be assessed by arranging two or more hot-film sensors with different orientations, the poor probe sensitivity in the transverse direction limits its uses to small angles; such sensor configuration is usually reserved for detecting shear reversal rather than its direction [5].

The electrodiffusion (ED) method measures the electrolysis reaction rate between an electrode flush-mounted to a wall and a redox couple contained in the flow. The method is in many ways similar to the hot-film anemometry, where the local *mass* transfer is measured instead of the heat transfer; the theory behind the two techniques overlaps in several aspects. Still, one major asset of the ED is the lack of heat loss to the wall, especially profitable in low convection flows. In their review of the wall shear stress produced by an impinging jet, Phares et al. [6] indeed concluded that the ED method provided 'the most accurate data close to the stagnation point', as the ones from hot-film probes suffered from strong discrepancies. While the cutoff frequency associated with ED probes is rather low, adequate post-processing can correct the attenuation and phase shift of the sensor response in highly unsteady flows. In particular, by considering that the reaction at the probe interface is governed by the convection–diffusion equation, one can take advantage of the so-called *inverse problem* to deal with the probe inertia. With this approach, the input wall shear rate is iteratively adjusted by solving the *direct problem* (i.e. the convection–diffusion equation) until the numerical data converge to the experimental ones. According to Rehimi et al. [7], such method allows to accurately correct the probe response in high amplitude unsteady flows, including the case of shear reversal. The authors have demonstrated that this method outmatches every other post-treatments in two-dimensional flows. Considering its success, we propose an enhancement of the method adapted to three-dimensional flows able to capture the wall shear rate magnitude along with its direction in any unsteady flow when using a three-segment probe. The method is validated numerically for flows subjected to periodic and stochastic variations of the wall shear rate.

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Nomenclature

D	diffusion coefficient
A	area of the probe
d	diameter of a circular ED probe
f	frequency
F	Faraday constant
c	concentration
C	dimensionless concentration, see (5)
C_0	concentration in the bulk solution
k_q	Lévêque constant
k^*	constant in (14), equals to Sh^*_{tot} when $Pe \to \infty$
I	limiting current
J	reaction rate
$\mathbf{J}$	sensitivity matrix, see (10)
N	number of unknowns in the inverse problem
$\boldsymbol{p}$	vector of unknowns in the inverse problem
Pe	Péclet number, see (5)
s	wall shear rate
S	dimensionless wall shear rate
Sc	Schimdt number (ν/D)
Sh	Sherwood number
Sh^*	modified Sherwood number ($ShPe^{-1/3}$)
Sr	Strouhal number, see (5)
$vecu$	velocity vector
t	time
T	Number of time steps
x, y, z	streamwise, normal and spanwise coordinates
X, Y, Z	dimensionless coordinates, see (5)

Greek symbols

α	wall shear rate direction
α_0	time average wall shear rate direction
β_α	amplitude on S for periodic flows, see (15)
β_S	amplitude on α for periodic flows, see (15)
ϕ	phase shift between periodic S and α
τ	dimensionless time, see (5) and (17)
ν	kinematic viscosity
ζ	attenuation ratio, see (16)

Subscripts

exp	experimental or 'true'
sob	Sobolík method
q	quasi-steady method
n	relative to S ($n = 0$) or α ($n = 1$)
m	segment m of a three-segment probe

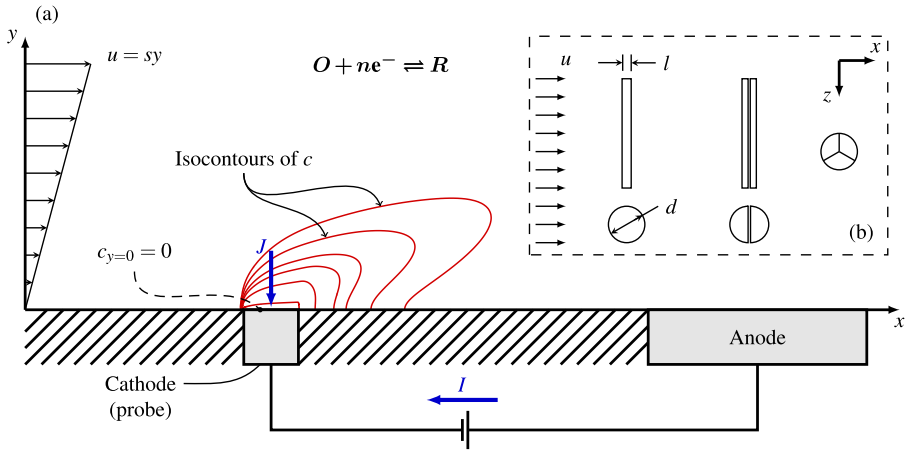

Fig. 1. Principle of the electrodiffusion (ED) method. (a) Under constant voltage, a time varying current $I(t)$ flows throw the loop cathode-solution-anode, generating concentration gradients at the electrodes surface from C_0 in the bulk to a concentration $c = 0$ at the probe–solution interface. (b) Typical probes, viewed from above. From left to right: single, double (sandwich) and three-segment probes.

1.1. Basic principles of the electrodiffusion method

The ED method measures the electric current I between the working electrode (probe) and a large counter electrode (Fig. 1). This current depends on convective and molecular transfer of active species (O–R) contained in the fluid, which are consumed by a fast electrochemical reaction. This process is initiated by imposing a constant voltage between the anode and the cathode, where a concentration gradient gradually builds up at the electrodes–solution interface. The current flowing through the electrodes and electrolyte is then a function of the solution supply and is directly related to the wall shear rate s under steady conditions. When streamwise convection is dominant, the relation $s \propto I^3$ typically holds over a certain voltage range, namely when the *limiting current* condition is achieved [8]; the reaction then occurs at the maximal rate possible and the concentration at the probe surface is essentially null ($c_{y=0} = 0$). During this process, mass transfer is manifested by an exchange of electrical charges between the O–R species. While this transfer is normally assured by three methods (migration, diffusion and convection), a non-reactive or background electrolyte is usually added in excess to the solution so as to limit migration effects. The divergence of the resulting Nernst–Planck equation, which dictates the mass transfer at an electrode, results in the general convection–diffusion equation in absence of migration:

$$\frac{\partial c}{\partial t} + \boldsymbol{u}\cdot\nabla c = D\nabla^2 c, \tag{1}$$

with D the diffusion coefficient. The relation between (1) and measures of $I(t)$ can also be derived from the Nernst–Planck equation. Under the assumption of a Nernst diffusion layer,[1] only the diffusion term remains and the flux or reaction rate J at the probe can be written as

[1] At the electrode surface, a stagnant layer of thickness δ_c is assumed; in other words, convection is neglected in this area, resulting in a frozen diffusion layer [9].