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Analysis of the exact solution for the evaporative convection problem and properties of the characteristic perturbations

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ABSTRACT

The exact solution for the Boussinesq approximation of the Navier – Stokes equations is analytically constructed to describe the joint flow of an evaporating viscous heat-conducting liquid and gas-vapor mixture in an infinite horizontal channel. The effects of thermodiffusion and diffusive heat conductivity in the gas-vapor phase are additionally taken into account in the governing equations and interface conditions. Possible flow types are classified with respect to the types of the boundary conditions for the vapor concentration on the upper solid wall of the channel. The importance of a solution of special type is that it gives a possibility to specify on the qualitative level the physical factors defining the basic flow mechanisms. The constructed solution has the group nature and allows one to describe the real flow regimes and a formation of patterns observed in the experiments. It describes three classes of flows: pure thermocapillary, mixed and Poiseuille flows according to the dominant force or interaction of effects. The stability of the two-layer flows of the liquid and gas-vapor phase is investigated for the equal values of the longitudinal temperature gradients on the channel walls. In this case the perturbations of the basic flow can lead to the formation of the vortex and thermocapillary structures. The influence of the thermal load intensity and gas flow rate on the type of the arising instability is studied. The evolution of the perturbations is investigated numerically.

1. Introduction

The theoretical study of the problems of convection with evaporation is often motivated by physical experiments. New physical experiments connected with the scientific projects of the European Space Agency (ESA MAP EVAPORATION) and Institute of Thermophysics SB RAS allows observing new phenomena in a fluid, which arise due to the gas flow related evaporation, as well as comparing the regimes, thermal patterns, intensity and flow topology under the conditions of gravity and microgravity [1,2]. The flow structures are determined by a set of dissimilar effects: the tangential stresses induced by a co-current gas flow, thermocapillary forces, type of external thermal load, thermophysical properties of working media, natural convection and evaporation/condensation effects. The combination of various effects significantly complicates the scientific problem. The action of each mechanism on the flow in the gas and liquid media should be investigated analytically and numerically. Results of the detailed study of the flow mechanisms is the basis for solving of the practical problems of thermostabilization and fluidic cooling, thermal controlling of highly efficient semiconductor equipment, thermal coating applications or drying processes. One of the important points in the physical experiments is to find the conditions which guarantee the stability of the basic state of the working media.

The modern experimental and theoretical study of the problems with evaporation is based on the results obtained in Refs. [3–11]. One of the main problems in the mathematical description of convective flows with the mass transfer at the interface is the correct formulation of the boundary conditions. On the basis of the treatments and basic foundations presented in the cited works of the XIX century the conditions at the interface were obtained in Refs. [12–19]. Different forms of these conditions are the consequences of some hypotheses on the interface and physical processes, which guarantee the fulfillment of the conservation laws. The long-wave approximation of the governing equations and boundary conditions was used in Refs. [20–23] to analytically and numerically study the convective flows of evaporating liquid media. The full statement of the problem including generalizations of the dynamic, heat and kinematic conditions is used in Refs. [19,24–26].

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Nomenclature

A, A_i longitudinal temperature gradients ($i = 1,2$)
a_m^i coefficients in the expressions of temperature ($i = 1,2$, $m = 1,2$)
b_m coefficient in the expressions of vapor concentration ($m = 1,2$)
C concentration of vapor in the gas-vapor phase, dimensionless
C_* saturated vapor concentration, dimensionless
c_j^i integration constants ($i = 1,2$, $j = 1, \ldots, 8$)
Ca capillary number, dimensionless
D coefficient of vapor diffusion, dimensional (m^2/s)
d_j^i coefficients in the expressions of temperature ($i = 1,2$, $j = 1,2,3$)
$\mathbf{g} = (0, -g)$ gravity acceleration vector, dimensional (m/s^2)
h_i layer thicknesses ($i = 1,2$), dimensional (m)
H mean interface curvature, dimensional ($1/m$)
k thermal conductivity, dimensional ($W/(m{\cdot}K)$)
K_j^i coefficients in the expressions of pressure ($i = 1,2$, $j = 1, \ldots, 8$)
L_j^i coefficients in the expressions of velocity ($i = 1,2$, $j = 3,4$)
L latent heat of evaporation, dimensional (J/kg)
M evaporation mass flux, dimensional ($kg/(m^2{\cdot}s)$)
p pressure, dimensional (N/m^2)
N_j^i coefficients in the expressions of temperature function ($i = 1,2$, $j = 2, \ldots, 7$)
R mass flow rate of the gas, dimensional ($kg/(m{\cdot}s)$)
R^* universal gas constant
Re Reynolds number, dimensionless
S_j coefficients in the expressions of concentration function ($j = 2, \ldots, 7$)
T temperature, dimensional (K)
$\mathbf{v} = (u, v)$ velocity vector, dimensional (m/s)
$\mathbf{x} = (x, y)$ Cartesian coordinates, dimensional (m)

Greek symbols

α coefficient defining the Soret effect, dimensional ($1/K$)
β thermal expansion coefficient, dimensional ($1/K$)
γ concentration coefficient of density, dimensionless
Γ notation of interface
δ coefficient characterizing the Dufour effect, dimensional (K)
ε coefficient in the linearized form of the Clapeyron-Clausius equation, dimensional ($1/K$)
ϑ component of temperature function, dimensional (K)
μ_0 molar mass of evaporating liquid, dimensional (kg/mol)
ν kinematic viscosity coefficient, dimensional (m^2/s)
ρ density, dimensional (kg/m^3)
σ surface tension, dimensional (N/m)
$\varkappa$ temperature coefficient of surface tension, dimensional ($N/(m{\cdot}K)$)
χ heat diffusivity coefficient, dimensional (m^2/s)
ϕ component of concentration function, dimensionless
Ω fluid layer

Subscripts and superscripts

0 reference flow parameters (specified at $T = T_0$)
i subscripts and superscripts to denote the functions and their components characterized the lower ($i = 1$) and upper ($i = 2$) layers
$*$ denotes characteristic value of functions

In the framework of different statements, manifold problems of evaporative convection were investigated. Some of them concern the stability of the liquid equilibrium [14,16,20,27–32] or the problem of a film flow [33–35]. Numerical modeling of the joint flows of evaporating liquids and gases is one way to forecast an outcome of experiments and to find specific mechanisms that determine the intensity of evaportion/condensation, character of different thermal, mechanical and structural effects. In addition, comparison of numerical and experimental results enables to modify a mathematical model or to evaluate the correctness of the problem statement. In Ref. [36] based on the numerical study the specific numerical criteria were obtained to choose an one-dimensional, partially two-dimensional or two-dimensional model for describing the evaporation of a water film falling on an inclined plate in a forced convection flow of humid air. The cases of the adiabatic, isothermal and uniformly heated plates being under constant heat flux were considered. As the criterion the air flow rate was chosen. Experimental and numerical investigations of mixed convection in a vertical rectangular channel were performed in Ref. [37]. A falling evaporating film was subjected to a laminar or turbulent airflow. Conditions of the most effective evaporative cooling were obtained. These conditions are the small heat fluxes on the channel walls and quite large airflow rates. The relation between the evaporation characteristics and thermal properties of a forced airflow was investigated in Ref. [38] experimentally and numerically in the framework of the $k - \varepsilon$ model. Dependence of evaporation rate on airflow rate, temperature drop and vapor concentration in air was studied in a wide range of the Reynolds numbers.

Modern theoretical methods of the convection theory include the formulation of the correct and physically meaningful mathematical models, construction, analysis and interpretation of the exact solutions for the governing equations, and investigation of the stability of the obtained solutions. The analysis results allow one to obtain the maps of stability, to define and explain critical mechanisms, leading to the stability loss. The exact solutions of the governing equations allow one to perform a thorough qualitative analysis of the effect of dissimilar factors defining the flow topology, evaporation intensity and mechanisms of instability. The correct physical interpretation of the exact solution and modeling results based on this solution can be used to estimate the adequacy and consistency of the problem statement.

In the classical sense an exact solution is a solution of the governing equations written in the form of perfect formulae, quadratures, series or special functions. According to [39] a set of exact solutions can be extended by the invariant and partially invariant solutions of the hydrodynamics equations of rank of 1 or 2 (see Refs. [40,41]). Most solutions of special type of the Navier – Stokes or Oberbeck – Boussinesq equations, including the considered in our paper generalization of the Ostroumov – Birikh solution [42,43], possess an invariant property with respect to some group of transformations admitted by these basic equations (our solution is the partially invariant solution of rank 1 and defect 3). Thus, the solutions allow one to effectively study the fundamental and secondary features of physical processes described with help of the Navier – Stokes equations, which imply the natural properties of space-time symmetry and symmetry of spatial fluid motion. Exact solutions (and the invariant or partially invariant ones) are important for testing numerical methods, analyzing singularities in solutions (in flows) and evaluating the approaches to the substantiation of approximative models. The asymptotic character of the Birikh type solution was justified in Ref. [44], where the thermocapillary gravitational convection in a long horizontal cavity was studied experimentally and numerically on the basis of a 2D problem with the boundary conditions, which model the conditions of the physical experiments.

One of the first exact solutions describing the flows in the system “liquid – liquid” with the mass transfer at the interface was presented in