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Transient analysis of a circular foil gage in a convective and radiative environment☆

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ABSTRACT

An analysis of a circular thin-foil gage is presented that includes transient effects, convective heat transfer, and an arbitrary time-varying boundary condition at the foil edge to account for fluctuations in cooling water temperature. The governing energy equation is solved using Laplace transform to obtain the temporal and spatial temperature distributions of the foil. Under constant temperature at the foil edge and constant thermal radiative heat flux, closed-form response curves for the gage under various modes of heat transfer are provided. Steady-state results are also presented as limiting cases.

1. Introduction

In 1953, Gardon [1] developed a circular thin-foil gage to measure thermal radiation and provided a simple steady state heat conduction analysis to describe the operation of the gage. Since then circular thin-foil gages, sometimes referred to as Gardon gages named in honor of Gardon, have been used extensively in fire research and testing [2], solar radiation measurements [3], and other aerospace applications [4]. A cross section of the gage is depicted in Fig. 1. The gage sensing element is made from constantan metal foil in the form of a circular disk and is attached to a water-cooled constant-temperature copper ring. When the incident heat flux strikes the foil, the energy absorbed by the foil diffuses radially toward the outer circumference of the foil and into the copper ring acting as a heat sink, causing a temperature difference between the foil center and the edge. The temperature difference, which is measured using a differential thermocouple, can be related to the incident heat flux.

Analytical studies have been reported in the literature to examine various aspects of the thin-foil gage operation and performance. Gardon [1] originally modeled the gage by treating it as a one-dimensional steady-state heat conduction through the foil in the radial direction to obtain the temperature distribution in the foil with an applied uniform thermal radiative heat flux on the foil. Malone [4] used a one dimensional steady-state formulation to analyze convective heat transfer to or from a radiantly heated foil and heat loss down the center wire. Ash [5] studied the transient response characteristics of thin-foil heat flux sensors under thermal radiative or convective environment. Kirchoff [6] provided a two-dimensional transient heat conduction analysis on the foil to study the effect of foil thickness on the gage response without considering heat convection. A one-dimensional transient heat conduction model was developed by Keltner and Wildin [7] to examine transient response of the gages. Borell and Diller [8], Kuo and Kulkarni [9], and Fu et al. [10] expanded the steady-state analysis of Gardon [1] by including convective heat transfer at the foil surface and provided calibration corrections for convective heat transfer. Recently, Fu et al. [11] applied the Duhamel theorem to study Gardon gage exposed to fast heat flux transients.

Here the analysis is expanded to include transient effects, convective heat transfer, and the time-varying heat-sink temperature to reflect fluctuations in cooling water temperature. However, we still limit our analysis to one dimension in the radial direction of the foil exposed to a constant radiative heat flux.

2. Formulation and analysis

Assuming no heat conduction loss along the copper wire at the center of the foil, no convective heat transfer on the backside of the gage, and constant thermophysical properties of the foil, the energy

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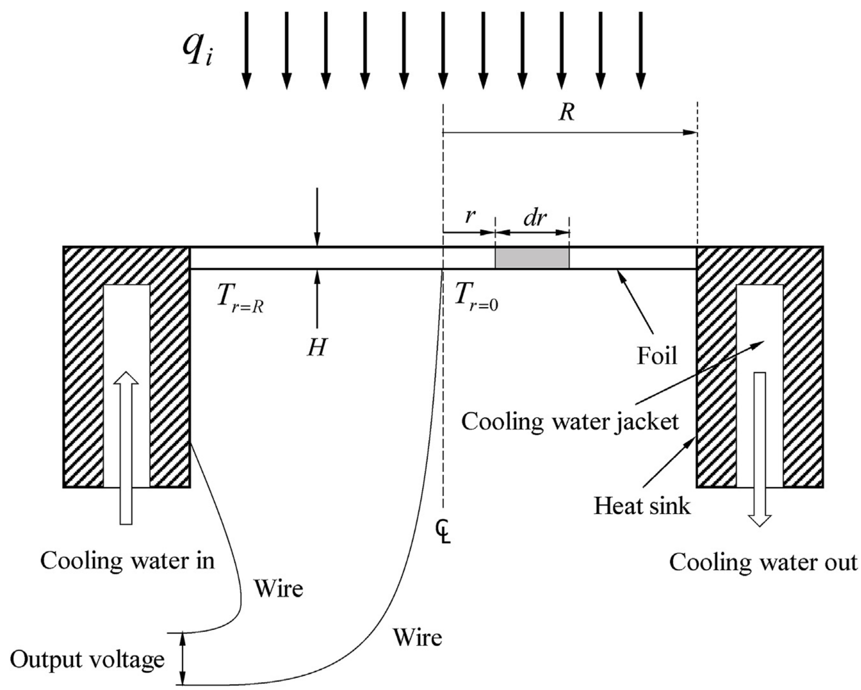

Fig. 1. Cross section of a thin-foil gage.

equation for the foil is

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{q_i}{kH} + \frac{h}{kH}(T_\infty - T) = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{1}$$

with the following initial and boundary conditions

I.C.: at $t=0$ $T = T_0$ for $0 \le r \le R$
B.C. 1: at $r=0$, $\partial T/\partial r = 0$ or T is finite
B.C. 2: at $r=R$ $T=f(t)$

The boundary condition (B.C. 2) at $r = R$ reflects an arbitrary time-varying heat-sink temperature.

Introducing the following dimensionless variables:

$\xi = r/R$ $\tau = \alpha t/R^2$ $\theta = T/T_0$

With $Nu = hR/k_f$, $Q^* = q_i R^2/kHT_0$, $\kappa = k_f/k$, $\delta = R/H$, and $Nu^* = \kappa\delta Nu$, Eq. (1) can be non-dimensionalized to

$$\frac{\partial^2\theta}{\partial\xi^2} + \frac{1}{\xi}\frac{\partial\theta}{\partial\xi} - Nu^*\theta + (Q^* + Nu^*\theta_\infty) = \frac{\partial\theta}{\partial\tau} \tag{2}$$

I.C.: at $\tau = 0$, $\theta = \theta_0$ for $0 \le \xi \le 1$
B.C. 1: at $\xi = 0$, $\partial\theta/\partial\xi = 0$
B.C. 2: at $\xi = 1$ $\theta = \theta_f(\tau) = f(\tau)/T_0$

Equation (2), together with the non-dimensionalized initial and boundary conditions, can be solved using Laplace transform. In the following, the Laplace transform and its inverse transform operators are represented by L and L^{-1} respectively, and we use $\hat{\theta}(\xi, s)$ to denote the transformed variable of $\theta(\xi, \tau)$ with respect to τ.

Taking Laplace transform of Eq. (2) with respect to τ,

$$L\left\{\frac{\partial^2\theta}{\partial\xi^2} + \frac{1}{\xi}\frac{\partial\theta}{\partial\xi} - Nu^*\theta + (Q^* + Nu^*\theta_\infty)\right\} = L\left\{\frac{\partial\theta}{\partial\tau}\right\}$$

$$\frac{\partial^2\hat{\theta}(\xi, s)}{\partial\xi^2} + \frac{1}{\xi}\frac{\partial\hat{\theta}(\xi, s)}{\partial\xi} - Nu^*\hat{\theta}(\xi, s) + \frac{(Q^* + Nu^*\theta_\infty)}{s} = s\hat{\theta}(\xi, s) - \theta(\xi, \tau = 0)$$

For clarity, we simply use $\hat{\theta}$ instead of $\hat{\theta}(\xi, s)$ in what follows.

$$\frac{d^2\hat{\theta}}{d\xi^2} + \frac{1}{\xi}\frac{d\hat{\theta}}{d\xi} - Nu^*\hat{\theta} + \frac{(Q^* + Nu^*\theta_\infty)}{s} = s\hat{\theta} - \theta_0$$

$$\frac{d^2\hat{\theta}}{d\xi^2} + \frac{1}{\xi}\frac{d\hat{\theta}}{d\xi} - (Nu^* + s)\hat{\theta} = -\left[\theta_0 + \frac{(Q^* + Nu^*\theta_\infty)}{s}\right] \tag{3}$$

B.C.1: at $\xi = 0$, $L\left\{\frac{\partial\theta}{\partial\xi}\right\} = \frac{\partial\hat{\theta}}{\partial\xi} = 0$
B.C.2: at $\xi = 1$, $\hat{\theta} = \hat{\theta}_f$

The general solution to Eq. (3) is

$$\hat{\theta}(\xi, s) = \frac{\left[\theta_0 + \frac{(Q^* + Nu^*\theta_\infty)}{s}\right]}{Nu^* + s} + c_1 J_0(i\xi\sqrt{Nu^* + s}) + c_2 Y_0(i\xi\sqrt{Nu^* + s}) \tag{4}$$

where c_1 and c_2 are constants and J_0 and Y_0 are the Bessel functions of the first and second kinds of zero order respectively. From B.C. 1, for $\hat{\theta}(\xi, s)$ to be finite at $\xi = 0$, $c_2 \equiv 0$ since $Y_0(0)$ approaches minus infinity.

From B.C. 2,

$$c_1 = \frac{\hat{\theta}_f(Nu^* + s) - \left[\theta_0 + \frac{(Q^* + Nu^*\theta_\infty)}{s}\right]}{(Nu^* + s)J_0(i\sqrt{Nu^* + s})}$$

Equation (4) then becomes

$$\hat{\theta}(\xi, s) = \frac{\theta_0}{(Nu^* + s)} + \frac{(Q^* + Nu^*\theta_\infty)}{s(Nu^* + s)} + \left\{\hat{\theta}_f - \frac{\theta_0}{(Nu^* + s)} - \frac{(Q^* + Nu^*\theta_\infty)}{s(Nu^* + s)}\right\}\frac{J_0(i\xi\sqrt{Nu^* + s})}{J_0(i\sqrt{Nu^* + s})} \tag{5}$$