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Advanced thermal impedance network for the heat diffusion with sources

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ABSTRACT

A new focus is given regarding the use of the quadrupole technique to solve heat diffusion problems with heat source in layered structures. The thermal impedances formalism is used that allows representing the heat transfer in such systems as impedances network. This approach offers a fast and accurate mean to calculate the average temperature of the heat source according to its spatial local and its transient evolution.

1. Introduction

The goal of this paper is to propose an extended application of the integral transforms technique [1,2] to the simulation of the heat transfer in layered composite materials with heat sources. Those sources can be located within different layers along the time, simulating for instance a moving heat source along one specific direction. Obviously, this study will envisage that the composite medium is constituted as a stack of layers having different thermal properties. It will be also accounted with the thermal boundary resistances between the layers. Several works have been already published that deal mainly with the two-layer composite medium [3–5]. Our purpose is to generalize the proposed thermal impedance network to composites involving several layers, whatever their number as the configuration presented in Fig. 1. The integral transforms method is at the foundation of the quadrupoles and thermal impedances network modelling. A specific approach, called the *QuadS* technique, has been already proposed for the 1D heat transfer in homogenous medium with a uniform heat source using the thermal impedances network with the source located at the central node of the elementary cell [6]. The main objective in implementing such a technique is to propose a fast and accurate method that allows calculating the average temperature at the location of the thermal disturbance.

Such a work finds its interest regarding the heat transfer modelling of advanced experimental techniques that have been developed during the last two decades that allow identifying the thermal properties of composite materials as the thermal conductivity, thermal diffusivity or the thermal boundary resistance at the interface between the layers. The thermoreflectance (TR) [7] and the flying spot infrared thermography (FST) [8,9] techniques consist in generating a thermal disturbance at several locations at the surface of the medium from a photothermal source (laser). The subsequent temperature change is measured from reflectance in the TR technique or from the proper emission of the medium in the infrared using the FST technique. According to the optical properties of the materials, this disturbance is generally considered as a heat source since the source penetration depth can be of the same order of magnitude as the source extension on the surface. Those techniques are now widely used since they provide a huge quantity of data that allows solving the inverse heat conduction problem in a very accurate way at the different scales of the heterogeneous medium. However, several issues remain critical regarding the simulation of the direct model with respect to the experimental configuration. Indeed, the characteristic dimensions of the thermal disturbance are generally very low with regards to the composite size in order to address the scale of the smallest heterogeneities. By the way, the discrete techniques (finite elements or volume elements) [10] require high computational resources and generate high computation times. Therefore, analytical approaches are preferred since they offer high accuracy and much less computation times and the proposed technique addressed in this paper intends to bring such an approach.

2. Illustration of the technique for heat diffusion in one layer with a localized heat source

2.1. Heat transfer model

As represented in Fig. 2, let us consider one layer with thickness e, specific heat C_p, density ρ and thermal conductivity tensor $\overline{\overline{K}} = (k_x,\ k_y,\ k_z)$. An instantaneous localized heat source term is defined as:

$$g(x, y, z, t) = \begin{cases} g_0\ \delta(t) & \text{for } 0 \le x \le a_x,\ 0 \le y \le a_y,\ 0 \le z \le e,\ t > 0 \\ 0 & \text{elsewhere} \end{cases} \tag{1}$$

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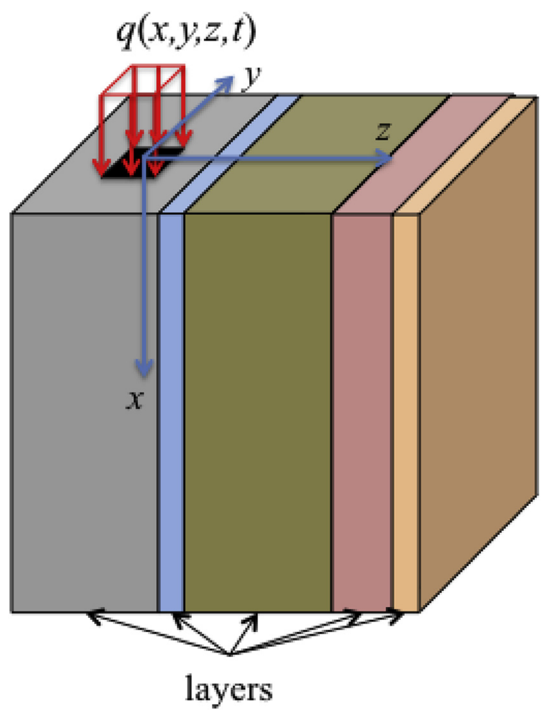

Fig. 1. Composite medium made of vertical layers with different thickness and thermal properties. A heat source q (only the location of the source is visible on the surface) is randomly applied on the surface.

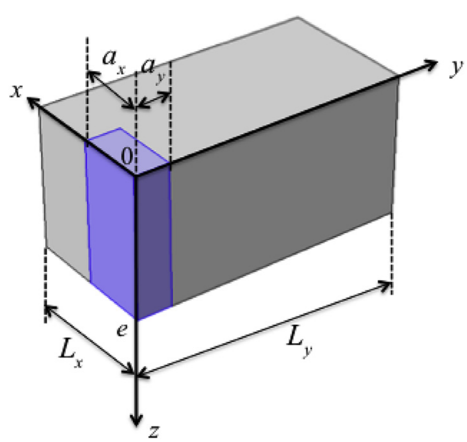

Fig. 2. Geometrical modelling for the heat source distribution in one layer. Thanks to the boundary conditions (null heat flux at the outer surfaces), only ¼ of the domain is represented.

The link between this test case and the experimental configuration presented in Fig. 1 will be established later in section 3. Using the geometrical parameters reported in Fig. 2, the mathematical equations for the heat transfer by conduction within the medium, describing the temperature $T(x, y, z, t)$, are:

$$\rho C_p \frac{\partial T}{\partial t} = k_x \frac{\partial^2 T}{\partial x^2} + k_y \frac{\partial^2 T}{\partial y^2} + k_z \frac{\partial^2 T}{\partial z^2} + g, \quad \text{for} \quad 0 < x < L_x, 0 < y < L_y, 0 < z < e, \quad \text{at} \quad t > 0 \tag{2}$$

The boundary conditions along x and y are:

$$\frac{\partial T}{\partial x} = 0, \quad \text{at } x = 0 \quad \text{and} \quad x = L_x, t > 0 \tag{3}$$

$$\frac{\partial T}{\partial y} = 0, \quad \text{at } y = 0 \quad \text{and} \quad y = L_y, t > 0 \tag{4}$$

The heat flux and temperature at $z = 0$ and $z = e$ are respectively denoted φ_0, T_0, φ_e and T_e, Finally, we assume a uniform initial condition as:

$$T = 0, \text{ for } \quad 0 \le x \le L_x, 0 \le y \le L_y, \ 0 \le z \le e, \text{ at } \ t = 0 \tag{5}$$

2.2. Integral transforms application

The Laplace transform is first applied on relations (1) to (5) with respect to the time variable as:

$$\theta(x, y, z, p) = \int_0^\infty T(x, y, z, t)e^{-p\,t}\mathrm{d}t \tag{6}$$

Then, relations (2) to (4) become:

$$\rho C_p p\theta = k_x \frac{\partial^2 \theta}{\partial x^2} + k_y \frac{\partial^2 \theta}{\partial y^2} + k_z \frac{\partial^2 \theta}{\partial z^2} + \tilde{g}, \quad \text{for } 0 < x < L_x, 0 < y < L_y, 0 < z < e \tag{7}$$

$$\frac{\partial \theta}{\partial x} = 0, \quad \text{at } x = 0 \quad \text{and} \quad x = L_x \tag{8}$$

$$\frac{\partial \theta}{\partial y} = 0, \quad \text{at } y = 0 \quad \text{and} \quad y = L_y \tag{9}$$

where $\tilde{g}(x, y, z, p) = g_0(x, y, z)$ denoted the Laplace transform of $g(x, y, z, t)$ (relation (1)). The transformed heat flux and transformed temperature at $z = 0$ and $z = e$ are respectively ψ_0, θ_0, ψ_e and θ_e. Let us note that in the case of a periodic variation of the source term at a given radial frequency ω, it is only required to replace p by $j\omega$ in the previous relations (7)–(9). Based on boundary conditions at $x = 0$, $x = L_x$, $y = 0$ and $y = L_y$, a cosine transform with respect to x and y coordinates is applied as:

$$\bar{\theta}(\alpha_m, \beta_n, z, p) = \int_0^{L_x}\int_0^{L_y} \theta \cos(\alpha_m x)\cos(\beta_n y)\mathrm{d}x\mathrm{d}y \tag{10}$$

With:

$$\alpha_m = \frac{m\pi}{L_x} \quad \text{and} \quad \beta_n = \frac{n\pi}{L_y} \tag{11}$$

Therefore, using the properties of the cosine transform with respect to the second derivative, relation (7) becomes:

$$\frac{\mathrm{d}^2\bar{\theta}}{\mathrm{d}z^2} - \left(\frac{p}{D_z} + \frac{D_x}{D_z}\alpha_m^2 + \frac{D_y}{D_z}\beta_n^2\right)\bar{\theta} + \frac{\bar{g}}{k_z} = 0, \quad \text{for } 0 < z < e \tag{12}$$

With:

$$\bar{g} = \begin{cases} g_0 a_x a_y, \ n = m = 0 \\ g_0 a_x \sin(a_y\beta_n)/\beta_n, \ m = 0, \ n \ne 0 \\ g_0 a_y \sin(a_x\alpha_m)/\alpha_m, \ m \ne 0, \ n = 0 \\ g_0 \sin(a_y\beta_n)\sin(a_x\alpha_m)/\beta_n\alpha_m, \ n \ne 0, \ m \ne 0 \end{cases}$$

In the relation (12) $D = k/(\rho\, C_p)$ denotes the thermal diffusivity. The heat flux and temperature at $z = 0$ and $z = e$ are respectively $\bar{\psi}_0$, $\bar{\theta}_0$, $\bar{\psi}_e$ and $\bar{\theta}_e$. As demonstrated in Ref. [1], the solution of relation (12) can be used in order to express the transformed heat flux and temperature at $z = 0$ according to the same quantities at $z = e$ as:

$$\begin{bmatrix} \bar{\theta}_0 \\ \bar{\psi}_0 \end{bmatrix} = \begin{bmatrix} A & B \\ C & A \end{bmatrix} \begin{bmatrix} \bar{\theta}_e \\ \bar{\psi}_e \end{bmatrix} - \begin{bmatrix} X \\ Y \end{bmatrix} \tag{13}$$

With:

$$A = \cosh(\delta_{m,n}e); \quad B = \frac{\sinh(\delta_{m,n}e)}{k_z\delta_{m,n}}; \quad C = k_z\delta_{m,n}\sinh(\delta_{m,n}e) \tag{14}$$