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Identification of space- and temperature-dependent heat transfer coefficient

Farzad Mohebbi*, Mathieu Sellier

Department of Mechanical Engineering, University of Canterbury, Private Bag 4800, Christchurch 8140, New Zealand

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ABSTRACT

The aim of this paper is to present a very efficient and accurate numerical algorithm to identify a variable (space- and temperature-dependent) heat transfer coefficient in two-dimensional inverse steady-state heat conduction problems involving irregular heat-conducting body shapes in the presence of Dirichlet, Neumann, and Robin boundary conditions. In this numerical method, a boundary-fitted grid generation technique (elliptic) is used to discretize the physical domain (heat-conducting body) and solve for the steady-state heat conduction equation by approximating the derivatives of the field variable (temperature) by algebraic ones. This paper describes a very accurate and efficient sensitivity analysis scheme to compute the sensitivity of the temperatures to variation of the variable heat transfer coefficient. The main advantage of the sensitivity analysis scheme is that it does not require the solution of adjoint equation. The conjugate gradient method (CGM) is used to reduce the mismatch between the computed temperature on part of the boundary and the simulated measured temperature distribution. The obtained results confirm that the proposed algorithm is very accurate, efficient, and robust.

1. Introduction

Inverse Heat Transfer Problems (IHTPs) are widely considered mathematically challenging problems. IHTPs are ill-posed and difficult to solve. Ill-posed problems are inherently unstable and very sensitive to noise. In other words, in such problems a small error in the input data can give rise to a large error in the solution [1–3]. Therefore, the development of efficient, accurate, and robust numerical schemes to solve IHTPs is of vital importance. Direct well-posed heat transfer problems are concerned with the determination of the temperature distribution over a heat-conducting body given that the boundary conditions, the thermo-physical properties, the geometrical configuration of the body, and the applied heat flux are all known. In contrast, the inverse heat transfer problem deals with the determination of the boundary conditions, the thermo-physical properties, the geometrical configuration of the heat-conducting body, and the applied heat flux from the knowledge of the temperature distribution on some part of the body boundary. Inverse heat transfer analysis has been extensively used to determine the thermo-physical properties such as the thermal conductivity and the convection heat transfer coefficient [4–28], the heat flux [16,25,29–33], and the boundary shape of bodies [34–40]. Other parameters involved in heat transfer problems are also estimated using an inverse analysis [41–43]. For example, in transient conduction–radiation heat transfer problems where the medium is participating, the extinction coefficient and the scattering albedo are parameters that affect the temperature distribution. In Chopade et al. [42], a combination of the lattice Boltzmann method (to solve the energy equation) and the finite-volume method (to compute the radiative information) with the particle swarm optimization is used to recover the extinction coefficient and the scattering albedo in a conducting–radiating planar participating medium.

The evaluation of the convection heat transfer coefficient is a difficult task because convection is a very complex phenomenon [44]. The convection heat transfer coefficient depends on many variables such as the geometry of the surface as well as the surface temperature, to name a few. The estimation of a variable convection heat transfer coefficient using an inverse analysis has also been reported [45–52]. In the literature, there exist some limitations on the proposed methods by different researchers to identify such a variable parameter. Some of these limitations can be summarized as follows:

- the applicability of the direct solver to rectangular or circular heated bodies only (using traditional finite-difference method) and inability to consider a general 2D domain.
- the inability to handle a variety of boundary conditions. Most of the boundary conditions in the literature include a constant temperature (Dirichlet boundary condition) or insulated case.

Thus a general methodology for a *general* 2D domain and boundary conditions considering a variable convection heat transfer coefficient with a high degree of accuracy is required. This paper deals with a two dimensional inverse steady-state heat conduction problem. The

* Corresponding author.
E-mail address: farzadmohebbi@yahoo.com (F. Mohebbi).

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Nomenclature

$d^{(k)}$	direction of descent at iteration k
$\dot{q}$	heat flux $\left(\frac{W}{m^2}\right)$
h	heat transfer coefficient $\left(\frac{W}{m^2.^\circ C}\right)$
Ja	Jacobian matrix
J	Jacobian of transformation
J	objective function
k_T	thermal conductivity of the solid body $\left(\frac{W}{m.^\circ C}\right)$
n	outward drawn unit vector
T	temperature (°C)
T_m	measured outer surface temperature (°C)
T_∞	ambient temperature (°C)
x, y	Cartesian coordinates in the physical domain

Greek symbols

α, β, γ	metric coefficients in 2-D elliptic grid generation
$\beta^{(k)}$	search step size at iteration k
Γ	boundary
$\gamma^{(k)}$	conjugation coefficient at iteration k
Ω	domain
ξ, η	Cartesian coordinates in the computational domain

Subscripts

i	grid index in ξ- direction
j	grid index in η- direction
M	number of grid points in the ξ- direction
N	number of grid points in the η- direction

Superscript

k	iteration number

objective of this study is to estimate a variable (space- and temperature-dependent) heat transfer coefficient in an irregular body. The convection heat transfer coefficient considered in this paper is a linearly space (boundary surface shape)- and temperature (boundary surface temperature)-dependent parameter. However, the linear form can be easily extended to other forms of dependency of the convective heat transfer coefficient on the space and temperature such as quadratic and cubic.

In the proposed numerical approach, an elliptic grid generation technique is used to generate a mesh over the irregular body and solve for the steady state heat conduction equation. The discretization in the computational domain is based on the finite-difference method, a method chosen for its simplicity and ease of implementation. The most innovative aspect of the numerical approach is its very efficient and accurate sensitivity analysis scheme, already introduced by the authors for other parameter estimation problems in heat transfer [16,24,25]. The sensitivity analysis scheme is formulated to compute the sensitivity of the temperatures to variation of the variable heat transfer coefficient. The conjugate gradient method is employed to minimize the difference between the computed temperature on part of the boundary and the simulated measured temperature. As will be shown, this numerical methodology does not require the solution of an adjoint problem. Explicit expressions for the sensitivity coefficients are derived which allow for the computation of the sensitivity coefficients in one single solve.

The proposed solution method introduced here is sufficiently general and can be employed for the estimation of a space- and temperature-dependent heat transfer coefficient applied on part of the boundary of a *general* two-dimensional region as long as the general two-dimensional region can be mapped onto a regular computational domain. Moreover, there is no limitation on the type of the boundary conditions. In other words, Dirichlet, Neumann, and Robin boundary conditions can be imposed on the domain boundary.

2. Governing equation

The mathematical representation of the steady-state heat conduction problem of interest here can be expressed as below (see Fig. 1)

$$\nabla^2 T = 0 \text{ in physical domain } \Omega \quad (1)$$

subject to the boundary conditions

$$\frac{\partial T}{\partial n} = -\frac{\dot{q}}{k_T} \text{ on boundary surface } \Gamma_1 \quad (2)$$

$$\frac{\partial T}{\partial n} = -\frac{h_i}{k_T}\left(T_{\Gamma_i} - T_{\infty i}\right) \text{ on boundary surface } \Gamma_i,\ i = 3,4 \quad (3)$$

Two different cases are considered for the boundary condition on the boundary surface Γ_2 which will be considered separately:

Case 1: The heat transfer coefficient is space-dependent (Fig. 1a):

$$\frac{\partial T}{\partial n} = -\frac{h_2(\Gamma_2)}{k_T}\left(T_{\Gamma_2} - T_{\infty 2}\right) \text{ on boundary surface } \Gamma_2 \quad (4)$$

where $h_2(\Gamma_2) = a_1 X_{\Gamma_2} + a_2 Y_{\Gamma_2} + a_3$.

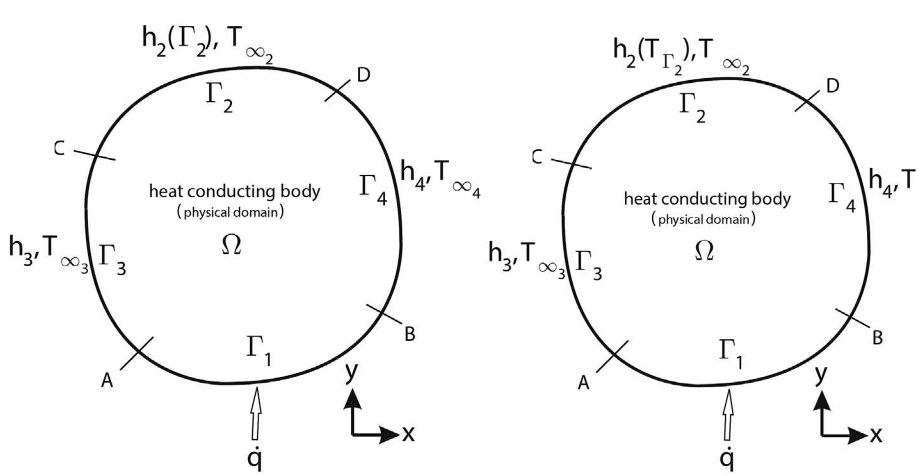

Fig. 1. Physical domain (solid body) subjected to convective heat transfer on surfaces Γ_i, $i = 2,3,4$ and heat flux $\dot{q}$ on surface Γ_1. The thermal conductivity of the body is k_T.