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Optimization-based design of easy-to-make devices for heat flux manipulation

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A B S T R A C T

In this work, we present a new method for the design of heat flux manipulating devices, with emphasis on their manufacturability. The design is obtained as solution of a nonlinear optimization problem where the objective function represents the given heat flux manipulation task, and the design variables define the material distribution in the device. In order to facilitate the fabrication of the device, the material at a given point is chosen between two materials with highly different conductivity. By this way, the whole device can be seen, in the large scale, as a metamaterial having a specific anisotropic effective conductivity. As an application example, we designed a heat flux inverter which was so simple that it could be hand-made. The performance of this device for heat flux inversion was experimentally tested, proving that it was more efficient than a more complex device designed using the classical transformation thermodynamics approach.

1. Introduction

Considering the major innovations enabled by the control of electromagnetic flux in electronics and communications [1] together with the analogies between electromagnetism and thermodynamics [2], the manipulation of heat flux is expected to lead to remarkable progress in thermodynamic applications. For instance, Chen and Lei [3] envisaged a dramatic enhancement of solar thermal collectors by using engineered thermal materials to concentrate the thermal flux.

These engineered materials are called metamaterials for having effective properties that goes *beyond* (*meta* in Greek) those found in nature, for instance negative apparent thermal conductivity [4]. In order to control heat conduction, a metamaterial must have a prescribed, spatially variable and generally anisotropic thermal conductivity. A device for heat flux manipulation consists of a body made of inhomogeneous metamaterial, which then has an inhomogeneous thermal conductivity distribution. In the literature, there are examples of devices designed and/or fabricated using metamaterials for different heat flux manipulation tasks: inversion [5–7], shielding [5–8], concentration [3,5–7], and cloaking [5,9]. Further, in our previous work, we designed metamaterial devices for combined shielding and cloaking [10] and combined concentration and cloaking [11]. In all these cases, the device serves to manipulate the heat flux in a given, say academic, way. Of course, they were effective to prove the efficiency and the potential of metamaterials, but their practical application needs further research and development.

We identify two main obstacles to the extension of the practical use of metamaterials. First, the classical approach for metamaterial design, that used in Refs. [3,5,8,9], is based on the thermodynamic transformation concept inherited from electromagnetism [12]. This methodology is, if not impossible, difficult to apply to general cases, for instance: 1) when either the metamaterial device or the region where it is embedded is not geometrically simple; 2) when the heat flux must be manipulated in general ways, or 3) when the external heat flux is not homogeneous.

To deal with arbitrary domains and boundary conditions, Dede [13] formulated the problem of metamaterial design as an optimization problem whose solution gives the metamaterial distribution. By this way, Dede determined the spatial variation of the orientation of the inclusions in a composite plate in order to minimize the thermal resistance of the plate. Later, Dede et al. [7] determined the orientation distribution in devices for heat inversion, concentration or shielding in a given portion of a plate. They represented each one of these tasks using a different objective function.

In our previous works [10,11], we have applied the optimization-based methodology for the design of thermal metamaterials for

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shielding and concentration (in both cases, combined with cloaking), respectively. We chose as thermal metamaterial a laminate of materials with high conductivity contrast, and we considered not only the orientation of the laminate but also the relative thickness of the layers at each finite element inside the device as design variables. Unlike Dede et al. [7], we defined a unique objective function for general manipulation tasks, capable of representing shielding, concentration, inversion and cloaking (as well as combination of these tasks) as particular cases. By this way, any so-defined heat manipulation task gives rise to a mathematically identical optimization problem. This general approach also serves for the purposes of this paper.

The other obstacle for real-life applications is the difficult fabrication of the metamaterial device, mainly because it has to be made with a precise inhomogeneous metamaterial distribution. This was circumvented by Vemuri et al. [14] by fabricating a device for heat concentration and cloaking using a homogeneous laminate that was arranged in two different orientations at each fourth of the device. By this way, they approached as well as possible (actually, quite poorly) the thermal conductivity distribution required to accomplish the given tasks.

So, the main goal of this paper is the design of easy-to-make devices for heat flux manipulation. Here, we postulate that the easiest-to-make device is that resembling a metamaterial only at the macroscale (that of the whole device) but is made of pure materials at the microscale. Specifically, at a given finite element in the device, a choice is made between two materials having sensibly different thermal conductivity. Further, this conductivity is not necessarily anisotropic as it is compulsory for guiding the heat flux using inhomogeneous metamaterials.

Like in our previous work using metamaterials [10,11], the desired heat flux manipulation task constitutes the objective function of a nonlinear constrained optimization problem, where the design variables define the material distribution throughout the heat flux manipulating device. Since the material at a finite element is either of two materials, the current problem strongly resembles a structural topology optimization problems [15]. Applications of topology optimization can also be found for heat conduction [16,17]. However, the current problem has crucial differences with classical topology optimization problems. First, the objective function in topology optimization is either the material volume or the compliance [15], linearly dependent on the design or the state variables, respectively. Meanwhile, the current objective represents the given task and is a highly nonlinear function of the design and the state variables. Secondly, to consider the material volume (either as an objective or as a constraint) is imperative for topology optimization but not for heat flux manipulation. Actually, we are purely concerned by the accomplishment of the task, obviating (at least, within the scope of this work) the minimization of material volume or the uniqueness of the solution.

Finally, we applied this optimization-based method to the design of a device for heat flux inversion. This is an extreme heat flux guidance problem, any other desired direction for the heat flux being an intermediate case. The so-designed device was so simple that we were able to hand-made it. We test this device for the experimental assessment of its efficiency, which was found to be better than that of Narayana and Sato's device [5], a considerable more complex device, designed on the base of the classical transformation thermodynamics approach and usually taken as reference to highlight the potential of metamaterials [1,2].

2. Heat conduction in a heterogeneous body

Let us consider the domain Ω in Fig. 1, made of a heterogeneous material, with boundary $\partial\Omega$ divided in two non-overlapping portions: $\partial\Omega_q$ (where the heat flux q_{wall} is prescribed) and $\partial\Omega_T$ (where the temperature T_{wall} is prescribed). In steady state, the heat conduction in Ω is governed by the equation

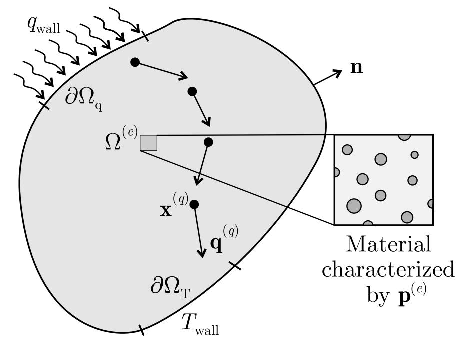

Fig. 1. Heat flux manipulation problem in the domain Ω where the effective properties at each sub-domain $\Omega^{(e)}$ depend on a set of parameters $\mathbf{p}^{(e)}$.

$$\mathrm{div}\,(\mathbf{q}) = s \quad \text{in } \Omega, \tag{1}$$

and the boundary conditions:

$$T = T_{wall} \quad \text{in } \partial\Omega_T, \tag{2}$$

$$\mathbf{q}\cdot\mathbf{n} = q_{wall} \quad \text{in } \partial\Omega_q, \tag{3}$$

where $\mathbf{q}$ is the heat flux vector field, s is the internal heat source, T is the temperature, and $\mathbf{n}$ is the unit vector normal to and pointing outwards $\partial\Omega$.

Assuming that the heat flux obeys the Fourier law, it is given by:

$$\mathbf{q} = -\mathbf{k}\,\mathrm{grad}\;T, \tag{4}$$

where $\mathbf{k}$ is the effective thermal conductivity, a second-order tensor.

Using the finite element method (FEM), the temperature field in Ω is approximated as follows:

$$T(\mathbf{x}) = N_j(\mathbf{x})T_j = \mathbf{N}(\mathbf{x})\cdot\mathbf{T} \quad \forall\ \mathbf{x} \in \Omega, \tag{5}$$

where N_j is shape function associated to the node j of the finite element mesh representing Ω, and T_j is the (unknown) temperature at this node. In the standard (Galerkin) FEM, the nodal temperature vector $\mathbf{T}$ is the solution of the algebraic system of equations

$$\mathbf{KT} = \mathbf{F}, \tag{6}$$

where $\mathbf{K}$ and $\mathbf{F}$ are the global conductivity matrix and the nodal heat flux vector, respectively, given by

$$\mathbf{K} = \int_\Omega \mathbf{B}^T\mathbf{kB}\,\mathrm{d}V, \tag{7}$$

$$\mathbf{F} = \int_\Omega s\mathbf{N}\,\mathrm{d}V + \int_{\partial\Omega_q} q_{wall}\mathbf{N}\,\mathrm{d}S, \tag{8}$$

with $B_{ij} = \partial N_j/\partial x_i$, such that $\mathbf{BT} =$ grad T.

The system of equation (6) is the FEM version of the heat conduction (1), subject to the boundary conditions (2) and (3), for the heat flux obeying the Fourier law (4). This is a classical FEM problem, whose details can be found for instance in the book of Zienkiewicz and Taylor of the basics of FEM [18].

Let us asume that Ω is a heterogeneous body. Specifically, the material is assumed to vary element-wise in the finite element mesh $\Omega = \Omega^{(1)} \cup \Omega^{(2)} \cup ...\cup\Omega^{(E)}$. At the finite element $\Omega^{(e)}$, the effective or material properties are functions of a finite number of parameters grouped in the vector $\mathbf{p}^{(e)} = [p_1^{(e)}, p_2^{(e)}, ..., p_M^{(e)}]$, as shown in Fig. 1. Examples of such parameters are the fiber orientation in fiber-reinforced polymers [19], the density and irregularity factors in materials with isolated inhomogeneities [20,21], the size of particles or beads in coating of dental implants [22,23], and the density in topological optimization [24]. So, the effective conductivity at the element $\Omega^{(e)}$ is

$$\mathbf{k}^{(e)} = \mathbf{k}(\mathbf{p}^{(e)}). \tag{9}$$

Further, made of contributions from all the elements of the mesh, the global conductivity matrix $\mathbf{K}$ is then