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Performance modeling of thermoelectric devices by perturbation method

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ABSTRACT

A nonlinear differential heat conduction equation for a leg of a thermoelement with temperature-dependent thermoelectric material properties is reduced to an integral equation. Its solution is obtained by a perturbation method in the form of a series in terms of powers of parameters proportional to Thomson and Joule heat. The first six coefficients of the series and the heat balance equations, i.e. the dependence of heat fluxes at the junctions on its temperatures and the electric current are found. The calculation results of main energy characteristics of thermoelectric devices (thermoelectric generators and thermoelectric coolers) is compared by five methods: exact solution, solution at constant thermoelectric properties taken at a mean temperature of hot and cold junctions, method of average parameters (thermoelectric material properties averaged over the relevant temperature range by integration), linear and quadratic approximations of the perturbation theory. It is shown that the accuracy acceptable in real-world applications (error less than 1%) is obtained for thermoelectric generators by the method of average parameters and for thermoelectric coolers - by quadratic approximation of the perturbation method.

1. Introduction

Thermoelectric generators (TEG) and thermoelectric coolers (TEC) are compact, reliable, noiseless and environmentally-friendly solid state heat engines that use electrons and holes as working fluid for direct conversion of thermal energy into electrical energy and vice versa. Despite their moderate energy efficiency, these devices found their specific niches in the markets and their development is continued towards materials improvement and cost reduction [1–5].

Choice of the schematic thermal and electrical diagrams of thermoelectric devices, their design and performance forecasting are based on the mathematical modeling of thermoelectric phenomena in semiconductors. Modern numerical methods of solving the governing equations of heat and electric charge transfer in solid body make it possible to calculate characteristics of any thermoelectric devices [6–15]. At the same time for many designs applied in practice the approximate analytical solutions of governing equations are widely used [11,16–20], including theirs use for describing some elements of complex multi-element thermoelectric devices at numerical modeling [21,22]. There are some reasons for this, for example:

- Any mathematical model is inevitably simplified, in particular since it uses approximated values of experimentally determined characteristics, for example, thermoelectric material properties. Therefore, the completeness of the problem mathematical description and accuracy of its solution should agree with the accuracy of the applied initial data.
- Thermoelectric devices often represent subsystems of complex systems including heat carriers, heat exchangers, radiators, multi-layer thermal and electrical insulators and interconnects, control devices, etc. Analytical solutions allow us to significantly simplify numerical calculations of such systems by reducing differential equations for electric charge and heat transfer in semiconductor legs to heat balance equation at junctions. In this case we have a combination of analytical and numerical methods.
- Analytical solutions not only reduce the time and laboriousness of the initial data preparation, calculations and analysis of their results but also afford the possibility of obtaining the universal conclusions on the impact of certain parameters on the characteristics of thermoelectric devices. For example, the use of a one-dimensional model of a thermoelement with temperature-independent properties made it possible to reveal a criterion of semiconductor quality, i.e. figure of merit$Z = \alpha^2/\rho\kappa$, where α is the thermoelectric power (Seebeck coefficient), κ is the thermal conductivity and ρ is the electrical resistivity [23].

The literature suggests a great number of various approaches to obtaining the analytical relationships for calculation of energy characteristics of thermoelectric devices: solving the differential heat conduction equations or making up energy balances at constant (temperature-independent) thermoelectric material properties [16,18,20–22,24–28], replacing variable properties with constant ones

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Nomenclature

Roman

A	coefficients of a perturbation series (dimensionless)
COP	coefficient of performance (dimensionless)
CP	cooling power (W)
EFF	efficiency (dimensionless)
$\boldsymbol{j}$	electric current density (Am^{-2})
l	length (m)
M	coefficient depending on figure of merit and mean temperature (dimensionless)
n	1, 2 and 3 denotes rectangular, cylindrical and spherical forms
P	power (W)
Q	heat flux (W)
$\boldsymbol{q}$	heat flux density (Wm^{-2})
S	cross sectional area (m^2)
T	temperature (K)
Z	figure of merit (K^{-1})

Greek

α	thermoelectric power (Seebeck coefficient) (VK^{-1})
β	dimensionless electric current
δ	dimensionless power
ΔT	temperature difference (K)
ε_T	dimensionless Thomson heat
ε_J	dimensionless Joule heat
θ	dimensionless temperature
κ	thermal conductivity ($Wm^{-1}K^{-1}$)
μ	dimensionless parameter
ν	product $\rho\kappa$ (V^2K^{-1})
ξ	dimensionless coordinate
ρ	electrical resistivity (Ωm)
τ	Thomson coefficient (VK^{-1})
φ	potential (V)
χ	dimensionless heat flux density
ψ	dimensionless cooling power

Abbreviations

TEG	thermoelectric generator
TEC	thermoelectric cooler

Subscripts

0	cold junction
1	hot junction
max	maximum

Superscripts

Sign ^	above a symbol means that the variable refers to a generalized one-dimensional case, the bar over the symbol denotes an average value and asterisk represents its dimensionless value (divided by the average value)

at mean temperature [11,29–33], using material properties averaged over temperature range by integration [23,34–36], obtaining solutions for special cases of material properties temperature dependences [19,37,38], averaging material properties over a hot or over a cold side of a thermoelement [37], determining energy characteristics by assessing the internal power losses in semiconductors [39–41], taking into account the fact that in most cases the thermoelectric effects are small as compared to the conduction heat transfer [42–44] or using an analogy between thermal and electric phenomena [11,29,45,46]. Disadvantages of the known methods are as follows: insufficient validity of analogy methods, the complexity of numerical algorithms and the laboriousness of preparing initial data, the lack of confidence in sufficient accuracy of methods of constant or average properties, and the impossibility to increase their accuracy.

Consequently, there arises the problem of selecting a method first of all to substantiate the applied approach under the minimum amount of simplifying assumptions, and secondly to obtain simple and at the same time rather accurate (admissible for practice) analytical relationships for thermoelectric devices modeling. This research applies the method of perturbations to solve the governing equations of heat and electric charge transfer in semiconductor with temperature-dependent properties with the aim to obtain analytical relationships for the heat balance equations. These equations significantly facilitate the modeling, including numerical, of thermoelectric devices energy characteristics and design parameters. The use of higher-order approximations makes it possible to obtain the results with the desired accuracy.

2. Thermoelectric phenomena

In an unevenly heated heterogeneous continuous medium, in addition to heat flux the temperature difference causes diffusion of the electric charge carriers, i.e. electric current. Electric current in turn is accompanied by heat generation or absorption. These phenomena are called thermoelectric. Thermodynamics makes it possible to establish a relation between the coefficients that characterize different thermoelectric effects, whereas kinetic theory allows us to find numerical values of the coefficients [23,47–49].

The generalized equations for electric charge and heat transfer relate the electric current density $\boldsymbol{j}$ and the heat flux density $\boldsymbol{q}$ with gradients of temperature T and potential φ [48]:

$$\boldsymbol{j} = -\frac{1}{\rho}(\alpha\nabla T + \nabla\varphi) \tag{1}$$

$$\boldsymbol{q} = -\kappa\nabla T + \alpha T\boldsymbol{j} \tag{2}$$

where thermoelectric material properties α, κ and ρ depend on temperature and coordinate.

In a special case, with no thermoelectric phenomena ($\alpha = 0$), the processes of electric charge and heat transfer become independent. Then from (1) and (2) there follow Ohm's law

$$\boldsymbol{j} = -\frac{1}{\rho}\nabla\varphi$$

and Fourier's law

$$\boldsymbol{q} = -\kappa\nabla T$$

In steady state conditions, the governing equations result from the principles of conservation of charge and energy [48].

$$\nabla\boldsymbol{j} = 0 \tag{3}$$

$$\nabla\ (\boldsymbol{q} + \varphi\boldsymbol{j}) = 0 \tag{4}$$

Energy flux density in (4) equals a sum of energy transferred by heat conductivity and energy carried by charged particles (electrons). By solving the differential equations (3) and (4) with the respective boundary conditions we can find the distributions of temperature and potential and then determine heat flux, electric current and other characteristics of the considered system.