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A new formulation for convection problems entailing multiple isothermal boundaries

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ABSTRACT

Many convection problems entail more than two isothermal boundaries. In previous work, a resistor-network model was proposed for this class, multi-temperature convection problems. A technique dubbed dQdT was also developed to obtain the paired convective resistances that characterize the thermal network of a multi-temperature convection problem. In the present paper an extension of the Newton law of cooling is proposed as a general formulation of multi-temperature convection in terms of multiple driving temperature differences. Most notably, the proposed formulation eliminates the need for an effective temperature difference. The formulation is characterized by functionality coefficients which give the relation between a heat transfer rate and one of the temperature differences. These coefficients can be obtained using the dQdT technique. The new formulation and the application of the dQdT technique are demonstrated for classical three-temperature convection problems. The connection between the extended Newton formulation and the resistor-network model of multi-temperature convection is also discussed. It is shown that dQdT can be used to determine the applicability of the resistor-network model of convection.

1. Introduction

Many convection problems entail exclusively isothermal and adiabatic boundary conditions. Furthermore, in many cases heat transfer occurs between more than two isothermal boundaries. A common example of this class, multi-temperature convection, is convective heat transfer in channels and annuli with isothermal walls. In this case, heat transfer is driven by more than one temperature difference. Accordingly, the Newton law of cooling must be reconciled with the presence of multiple temperature differences. Traditionally, an effective temperature difference is constructed to formulate multi-temperature convection. In previous work [1–3], it was demonstrated that using a single effective temperature difference to formulate multi-temperature convection can be problematic as it leads to non-physical peculiarities in the solution. See for example the singularities in the temperature-dependent Nusselt numbers reported by Hatton & Turton [4] for the asymmetric Graetz problem (forced convection) and the discussion of those results by various researchers [1,5,6]. Other examples include the temperature-dependent Nusselt numbers presented by Mitrović & Maletić [7] and Coelho & Pinho [8] for forced convection in an annulus with isothermal walls, and the difficulties reported by Roeleveld et al. [9] in developing correlations for the wall Nusselt numbers of free convection in a vertical channel with asymmetrically heated isothermal walls.

Recently a resistor-network model has been developed for multi-temperature convection [1,2,10]. In this model, the isothermal boundaries are represented by nodes at the corresponding boundary temperatures. It has been shown that using the resistor-network model, shortcomings of the existing approach can be addressed. See for example reference [1]. A technique called dQdT has also been developed [2] to evaluate the paired resistances of the resistor-network model of convection. Nevertheless, as will be shown, the resistor-network model is only applicable under certain conditions. In the present paper, an extension of the Newton law of cooling is developed as a general formulation of multi-temperature convection. The proposed formulation eliminates the need for an effective temperature difference, by formulating the multi-temperature convection problem in terms of multiple temperature differences. The connection between the proposed formulation and the earlier developments, namely the resistor-network model and the dQdT technique, is discussed. It is shown that dQdT can be used to obtain the coefficients of the proposed formulation. It is further shown that dQdT can be utilized to determine the applicability of the resistor-network model.

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2. The Newton formulation

According to the Newton law of cooling, the rate of convective heat transfer is proportional to a driving temperature difference. This temperature difference is usually the difference between surface and fluid temperatures. The Newton law of cooling is expressed mathematically by introducing a proportionality coefficient, known as the heat transfer coefficient, h, as shown in Equation (1). In this equation, Q_1 is the heat transfer rate at the surface, A is the surface area, and T_1 and T_0 denote the surface and fluid temperatures respectively.

$$T_1 \tag{1}$$

Although h is introduced as a proportionality coefficient, it is not necessarily a constant. In other words, the relation between the heat transfer rate and temperature difference is not necessarily linear. Any nonlinearity in Q_1 with respect to T_1 or T_0 is contained in h.

Further note that tacit in the Newton formulation is the assumption that convective heat transfer occurs in a setting with *two* representative temperatures, i.e. driven by a single driving temperature difference. A standard example is forced convection in flow over an isothermal flat plate, shown in Fig. 1. In this case, the free-stream temperature, T_0, and the surface temperature, T_1, are the two representative temperatures; heat transfer is driven by $T_1 - T_0$. Note that T_0 and T_1 constitute the thermal boundary conditions of the problem.

Consider now convective heat transfer in the parallel-plate channel shown in Fig. 2. The flow enters the channel at a uniform temperature, T_0, and the channel walls are maintained at uniform temperatures T_1 and T_2. The thermal boundary conditions therefore entail the set of three independent temperatures: $\{T_0, T_1, T_2\}$. Heat transfer in the channel is governed by these three boundary temperatures. More specifically, heat transfer is driven by three temperature differences: $\Delta T_{10} = T_1 - T_0$, $\Delta T_{20} = T_2 - T_0$ and $\Delta T_{12} = T_1 - T_2$. There are also three heat transfer rates of interest: the heat transfer rate from the walls, Q_1 and Q_2, and the rate of total heat transfer from the fluid; $Q_0 = -(Q_1 + Q_2)$.

In order to formulate the *three-temperature* problem shown in Fig. 2, Equation (1) must be reconciled with the presence of multiple temperature differences. This is customarily done by constructing a single effective temperature difference through a combination of the independent (and sometimes dependent) temperatures. In internal-flow problems, the mean fluid temperature, T_m, is usually used to represent the fluid flow. The choice of T_m in two-temperature forced convection problems (e.g. flow in an isothermal pipe) is advantageous because it leads to a constant Nusselt number in the fully developed region. Note that T_m is a dependent variable. T_m is also used to represent the flow in multi-temperature problems. See for example reference [4]. The rate of heat transfer from the channel walls can hence be expressed as shown in Equation (2).

$$T_0 \tag{2}$$

For free convection in isothermal passages, following the seminal work of Aung [11] on vertical channels, the flow is usually represented by T_0. The wall heat transfer rates are hence expressed as shown in Equation (3) (e.g. Ref. [9]).

$$Q_i = h_i\,A\,(T_i - T_0)(i = 1{,}2) \tag{3}$$

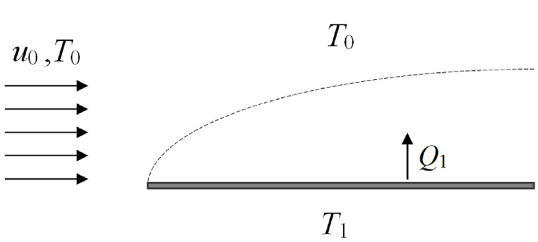

Fig. 1. Forced convection in flow over an isothermal flat plate.

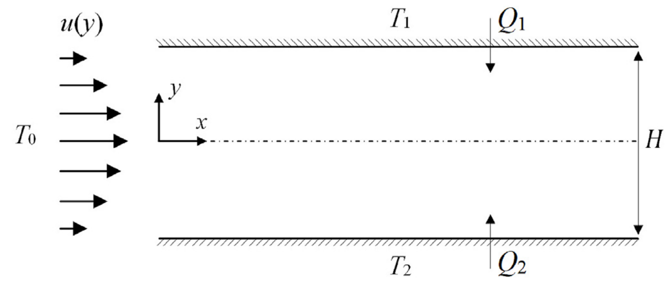

Fig. 2. Convection in hydrodynamically developed, laminar, constant-property flow in a parallel-plate channel with isothermal walls — The asymmetric Graetz problem.

Similarly, using the mean wall temperature, $T_{wm} = (T_1 + T_2)/2$, to represent the channel walls and construct an effective temperature difference, the rate of total heat transfer from the fluid can be expressed as shown in either Equation (4) or Equation (5).

$$Q_0 = -(Q_1 + Q_2) = h_0(2A)(T_m - T_{wm}) \tag{4}$$

$$Q_0 = -(Q_1 + Q_2) = h_0(2A)(T_0 - T_{wm}) \tag{5}$$

As mentioned earlier, using an effective temperature difference to formulate multi-temperature convection problems can be problematic. In the case of forced convection in asymmetrically heated passages, this approach leads to singularities in and temperature-dependence of the Nusselt number. See Refs. [1,3,5,6,12,13] for detailed discussion of these peculiarities. In addition, Equation (1) does not reflect the physics of a multi-temperature problem in full. Alternatively, $\{Q_i\}$ may be formulated in terms of multiple driving temperature differences.

3. The Newton formulation extended

It is known from the mathematics of the problem that heat transfer in the configuration shown in Fig. 2 is influenced by all the three independent temperatures, i.e. by the set of boundary temperatures, $\{T_i\}$. More specifically, Q_0, Q_1 and Q_2 are all functions of $\{T_i\}$. Equation (6) is the mathematical expression of this observation (for a given combination of geometry, fluid properties and flow field).

$$Q_i = Q_i(\{T_i\}) \tag{6}$$

As suggested by the Newton formulation (Equation (1)), it is the *difference* between the boundary temperatures that drives heat transfer. The functional relation between $\{Q_i\}$ and $\{T_i\}$ for a given combination of geometry, fluid properties and flow field can accordingly be rewritten as shown in Equation (7) to emphasize the role of the temperature differences. Note that obtaining Equation (7) from Equation (6) entails merely a linear change of variables.

$$Q_i = Q_i(\{\Delta T_{ij}\}) \tag{7}$$

To recast Equation (7) into a form analogous to Equation (1), Q_i can be expanded in form of the summation shown in Equation (8), with ΔT_{ij} explicitly factored out of each term. Any nonlinearity with respect to $\{T_i\}$ is contained in the coefficients, $\{C_{ij}\}$.

$$Q_i = \sum_j C_{ij}\Delta T_{ij} = \sum_j C_{ij}(T_i - T_j) \tag{8}$$

Note that only *independent* temperatures appear in Equation (8). The coefficient C_{ij} characterizes the relationship between Q_i and the driving temperature difference ΔT_{ij}. It is hence appropriate to call C_{ij} a "functionality coefficient". The subscript ij is introduced to indicate that C_{ij} corresponds to a specific temperature difference, ΔT_{ij}.

Applying Equation (8) to the three-temperature setting of Fig. 2, the rate of heat transfer at the walls can be written as: