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# Heat transfer enhancement in a modified straight fin of rectangular profile with enlarged round base and correlated reduced round tip

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ABSTRACT

The present study addresses the potential heat transfer enhancement in a modified straight fin of rectangular profile with an enlarged round base compensated by a reduced round tip when compared against the standard straight fin of rectangular profile. The straight fins of rectangular profile share the profile area (volume) and are subject to two controlling parameters: the transverse Biot number and the aspect ratio. In the common straight fin of rectangular profile, a small fraction of material approximately equal to two thirds of the semi-thickness squared is removed from the tip region and is attached to the base region. Thereby, the tenuous material migration creates the modified straight fin of rectangular profile with an enlarged round base and a reduced round tip.

Owing that the modified straight fin with round base and round tip possesses a pronounced irregular profile area, the one-dimensional modeling with the simplistic quasi one-dimensional heat conduction equation is inappropriate. Switching to the two-dimensional modeling, the governing two-dimensional heat conduction equation in Cartesian coordinates cannot be solved analytically and forcibly has to be done numerically with the potent finite element method. The two-dimensional temperature distributions and the companion heat transfer rates are computed for the two distinct straight fins with rectangular profile using realistic combinations of the transverse Biot number and the aspect ratio normally used in engineering practice. At the end, the maximum heat transfer enhancement granted by the modified straight fin with round base and round tip having small-to-moderate aspect ratios can reach a favorable 12% margin.

## 1. Introduction

Fins or extended surfaces are used extensively in industry to invigorate the heat dissipation from heated solid bodies to cold surrounding fluids for a fixed temperature difference or contrarily to decrease the temperature difference between the heated solid bodies and cold surrounding fluids (Kraus et al. [1]). The use of fins is found in applications of heat transfer engineering, such as internal combustion engines, air conditioning, refrigeration, compressors, heat exchangers, electrical equipment and electronic components, where engineers have succeeded in designing, fabricating, testing and operating smaller and efficient heat transfer devices on a continuous basis throughout the years. In general, fins are classified as straight or longitudinal and annular or circular with various profiles.

The most common types of straight fins have rectangular, triangular, trapezoidal, convex parabolic and concave parabolic profiles and are discussed in Ref. [1]. However, among them, the straight fins of rectangular profiles and the straight fins of trapezoidal profiles are customarily employed in applications of heat transfer engineering.

Quantification of the heat transfer rate from a heated solid through any bundle of fins to a neighboring fluid requires precedent knowledge of the temperature distribution in a single fin. In practice, the usage of fins is beneficial when the thermal conductivity $k$ is high and mean convection coefficient $\bar{h}$ is low, as is often the case for air, gases and water vapors in natural convection or even forced convection with low-to-moderate velocities.

Regardless of the shape, fins are longer than they are thick. Because of the geometric feature, it had been common and fairly accurate to assume that the temperature in fins varies primarily in the axial direction and the temperature is essentially uniform in the cross section [1]. This intuitive argument gives rise to the so-called quasi one-dimensional approximation. Obviously, this approximation is indeed a "rule-of-thumb" approach, which does not take into consideration relevant thermo-physical elements, such as the ratio of the interior conductive resistance $R_k$ to the exterior convective resistance $R_v$ and the aspect ratio (length to half-thickness). Fundamentally, the validity of

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**Nomenclature**

| | |
|---|---|
| $A_b$ | cross-sectional area at the base |
| $A_p$ | profile area |
| $Bi_L$ | length-based Biot number, |
| $Bi_t$ | transverse Biot number, $\frac{\overline{h}\,t}{k}$ |
| $E$ | heat transfer enhancement |
| $\overline{h}$ | mean convection coefficient |
| $k$ | thermal conductivity |
| $L$ | length |
| $q$ | heat flux |
| $Q$ | heat transfer rate |
| $R$ | radius of quadrant |
| $t$ | semi-thickness |
| $T$ | temperature |
| $T_b$ | base temperature |
| $T_f$ | fluid temperature |
| $T_m$ | mean temperature |
| $V$ | volume |
| $x$ | axial coordinate |
| $X$ | dimensionless axial coordinate, $\frac{x}{L}$ |
| $y$ | transverse coordinate |
| $Y$ | dimensionless transverse coordinate, $\frac{y}{t}$ |
| $w$ | depth |

*Greek symbols*

| | |
|---|---|
| $\theta$ | dimensionless temperature, $\frac{T-T_f}{T_b-T_f}$ |
| $\delta$ | thickness of straight rectangular fin, *2t* |
| $\Gamma$ | boundary of the modified straight rectangular fin |
| $\Omega$ | domain in the modified straight rectangular fin |

*Subscripts*

| | |
|---|---|
| Rect | rectangular |
| rect-curv | rectangular curved |

the quasi one-dimensional approximation is controlled by the interplay between $R_k$ and $R_v$ by means of the transverse Biot number based on the half-thickness, namely the relation $Bi_t = \frac{R_k}{R_v} = \frac{\overline{h}\,t}{k} << 1$in conjunction with moderate-to-large aspect ratios $\frac{L}{t} >> 1$.

From heat conduction theory (Arpaci [2]), the rigorous analysis of fins demands a full two-dimensional model, rather than the simplified quasi one-dimensional model. To the authors' knowledge, the two-dimensional treatment of straight fins has been mostly done for the trapezoidal profile with straight sides (wherein the rectangular and triangular profiles are particular cases). Representative references of these efforts are those of Lau and Tan [3], Chung et al. [4] Chen [5], Huang and Shah [6], Razelos and Georgiou [7], Aziz and Nguyen [8], Juca and Prata [9], Yeh [10], Onur [11], Kang and Look Jr. [12], Singh et al. [13], Xia and Jacobi [14], Marin et al. [15], and Malik and Rafiq [16].

In general, the temperature in a straight fin of rectangular profile descends monotonically from the base to the tip. Because the straight fin has its lowest temperature at the tip, removing a small amount of material from the tip region will not affect the fin performance significantly. On the other hand, adding a small material near the base region increases the contact area between the fin and the primary solid body and consequently augments the heat transfer rate. The primary objective of the present study is to examine the attributes for heat transfer enhancement that a modified straight fin of rectangular profile with an enlarged round base and a proportionally reduced round tip may bring forth. This particular profile is not included in the inventory of straight fins described in Reference [1]. For comparison purposes, the logical baseline case to be employed embodies a standard straight fin of rectangular profile sharing the same profile area (volume). The mathematical formulation of the two dissimilar straight fins of rectangular profile will begin with a general two-dimensional heat conduction equation. Due to the marked irregular profile area in the modified straight fin with enlarged round base and commensurate reduced round tip, the two-dimensional temperature solution has to be obtained numerically with the potent Finite Element Method.

## 2. Formulation for the straight fin of rectangular profile

Consider the straight fin of rectangular profile with dimensions: length $L$, thickness $\delta$ and depth $w$ sketched in Fig. 1. It is presumed that there is no thermal contact resistance at the fin base, the thermal conductivity of the material $k$ is constant or nearly independent of temperature, the convection coefficient $h$ is suitably represented by the mean convection coefficient $\overline{h}$. In addition, the fin base is maintained at a high temperature $T_b$ and a fluid flows across the fin surface with a low temperature $T_f$. Radiation effects to the fluid is neglected.

In general, the governing steady, two-dimensional heat conduction equation framed in Cartesian coordinates is

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \tag{1}$$

Exploiting geometric and thermal symmetry in the straight fin of rectangular profile, the boundary conditions in the upper part are:

i) Prescribed temperature at the base:

$$T = T_b, \quad x = 0, \quad \text{all } y \tag{2}$$

ii) Heat convection at the tip:

$$-k\frac{\partial T}{\partial x} = \overline{h}\,(T - T_f), \quad x = L, \quad \text{all } y \tag{3}$$

iii) Zero heat flux at the horizontal mid-plane

$$q = \frac{\partial T}{\partial y} = 0, \; y = 0, \; \text{all } x \tag{4}$$

iv) Heat convection along the exposed surface:

$$-k\frac{\partial T}{\partial y} = \overline{h}\,(T - T_f), \; y = t, \; \text{all } x \tag{5}$$

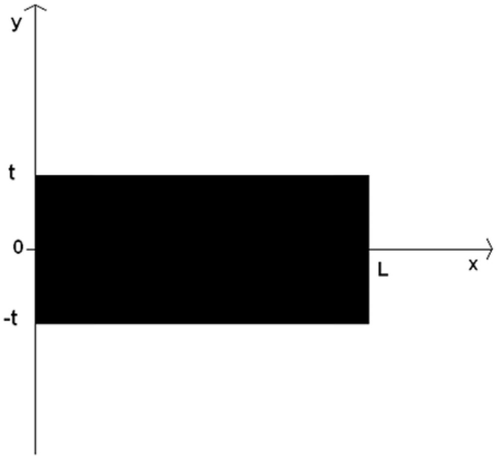

**Fig. 1.** Straight fin of rectangular profile.