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Meshfree numerical simulation of free surface thermal flows in mould filling processes using the Finite Pointset Method

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ABSTRACT

The purpose of this work is to carry out a meshfree implementation for the numerical simulation of two-dimensional transient incompressible free surface flows coupled with heat transfer. The Finite Pointset Method is applied in order to solve the involved partial differential equations where the corresponding classical or strong formulation is directly used instead of the corresponding weak form as needed for some other meshfree approaches. The incorporation of the boundary conditions is done in a direct and simple manner. The simplicity and efficiency of this numerical method are demonstrated on two complex two-dimensional mould filling processes.

Nomenclature

A_i PDE coefficient [·]
$\mathbf{B}$ PDE coefficient [·]
C_i PDE coefficient [·]
F PDE coefficient [·]
G PDE coefficient [·]
J Auxiliary matrix [·]
L Auxiliary matrix [·]
O Auxiliary matrix [·]
P Differences matrix [·]
P_{xy} Differences matrix [·]
$\mathbf{R}$ Auxiliary matrix [·]
T Fluid temperature [°C]
T_c Cold/reference temperature [°C]
T_h Hot temperature [°C]
T_{in} Inlet temperature [°C]
T_∞ Ambient temperature [°C]
h_w Heat transfer coefficient at walls [W m^{-2}°C^{-1}]
k Thermal conductivity [W m^{-1}°C^{-1}]
k_s Thermal conductivity at free surface [W m^{-1}°C^{-1}]
k_w Thermal conductivity at walls[W m^{-1}°C^{-1}]
$\mathbf{n}$ Boundary normal vector [·]
p Flow pressure [Pa]
$p_k(\bar{\mathbf{x}})$ Linear independent functions [·]
$\mathbf{t}$ Boundary tangential vector [·]
$\mathbf{v}$ Flow velocity vector [m s^{-1}]
$\mathbf{v}_0$ Initial velocity [m s^{-1}]
$\tilde{\mathbf{v}}$ Temporal flow velocity [m s^{-1}]
w Weight function [·]
$\mathbf{x}$ Arbitrary fluid point [m]
W Weight matrix [·]
$b_k(\mathbf{x})$ Coefficients in Taylor series [·]
$\mathbf{b}$ Unknowns vector [·]
c Specific heat [J kg^{-1}°C^{-1}]
$\mathbf{e}$ Truncation error vector [·]
f Arbitrary function value [·]
$\tilde{f}$ Approximated function value [·]
$\mathbf{f}$ Function value vector [·]
$\mathbf{f}_b$ Distributed body force [m s^{-2}]
$\mathbf{g}$ Gravitational acceleration vector [m s^{-2}]
h Smoothing length in w [m]
$h_{i,k}$ Spatial differences [·]
h_s Heat transfer coefficient at free surface [W m^{-2}°C^{-1}]
τ Viscous stress tensor [Pa] at free surface [W m^{-2}°C^{-1}]
$\mathbf{x}^k$ Particle position at k-th iteration [m]
$\mathbf{x}_i$ i-th particle position [m]
Γ_d Dirichlet boundary [·]
Γ_n Neumann boundary [·]
Φ Shape function [·]
Ω A given fluid domain [·]
β Coefficient of thermal expansion [°C^{-1}]
γ Weight function parameter [·]
ω Angular velocity [s^{-1}]
μ Fluid dynamic viscosity [Pa s]
ν Fluid kinematic viscosity [m^{2}s^{-1}]
φ Prescribed normal derivative [·]
ρ Fluid density [kg m^{-3}]
Δt Time step [s]
Δx_i Spatial differences [m]

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ξ_1	Auxiliary matrix $[\cdot]$	Δy_i	Spatial differences $[m]$
ξ_2	Auxiliary matrix $[\cdot]$	Δz_i	Spatial differences $[m]$
$\partial\Omega$	Boundary of fluid domain $[\cdot]$	ψ_j	$j-$th unknown component $[\cdot]$
$\frac{D}{Dt}$	Material derivative $[s^{-1}]$	$\widetilde{\Psi}$	Unknowns vector $[\cdot]$
∇	Gradient operator $[m^{-1}]$	Re	Reynolds number $[\cdot]$
Δ	Laplace operator $[m^{-2}]$		

1. Introduction

Heat transfer coupled with fluid flow arises in many engineering applications, in particular for continuous casting processes, metal forging and forming through mould filling processes in the foundry industry. Numerical methods for partial differential equations like Finite Difference Methods, Finite Volume Methods and Finite Element Methods have been widely used for such purposes, among others mesh-based methods. However, mesh-based techniques need to use remeshing approaches in order to simulate problems with rapidly changing geometries as those involving free surface flows, which are both computational and economically expensive for these kind of problems. These drawbacks arise directly from the need of using predefined meshes to solve the governing equations [1]. Recently meshfree or meshless methods have been developed as alternative to overcome part of the difficulties arising when mesh-based methods are used. They are classified in two main groups according to the type of equations on which they are based [2,3].

One group of meshfree methods are based on the weak-form of the corresponding partial differential equations and they are characterized by being stable and accurate, therefore they naturally satisfy the imposed Neumann boundary conditions. However, the mandatory numerical integration in this kind of methods makes them computationally expensive. Moreover, they are not completely meshfree since they require local or global meshes for integrating the derived matrix system from the weak-form on the problem domain which constitutes a drawback when they are used in problems involving high deformations. The most common examples of such methods include the Element Free Galerkin Method (EFG), the Reproducing Kernel Particle Method (RKPM), the Diffuse Element Method (DEM), the Meshless Local Petrov-Galerkin Method (MLPG), the Meshless Boundary Element Method (MBEM), the Meshless Finite Volume Particle Method (MFVPM) and the Natural Element Method (NEM). The second group of meshfree methods are based on the strong-form of the corresponding partial differential equations and they are characterized by being truly meshfree since they do not require any kind of meshing during the solving process, moreover they are easy to implement and computationally efficient. These facts make them especially attractive for the modeling of problems involving highly changing geometries such as problems with free surface flows. Nonetheless, many of them are unstable and less precise than weak-form methods when Neumann boundary conditions are involved. The most common examples of these kind of methods include the Smoothed Particle Hydrodynamics (SPH), Finite Pointset Method (FPM), Finite Point Method and the Radial Basis Function Methods (RBF) [1–12].

In the context of meshfree methods, the starting point was SPH proposed by Monaghan and Gingold in Refs. [13] and [14] for astrophysical applications and it has also been taken as a common and practical method applied to predicting complex fluid flows and mechanical processes, in particular for casting processes [15–17], due to its ability to model the behaviour of complex free surfaces and their ability to tolerate high levels of deformation as well as tracking the deformation history [18–22]. Nonetheless, since the development of the original version, SPH suffered instability, inconsistency and difficulties in proper treatment of the boundary conditions so that over the next years many improvements were incorporated to the original SPH formulation. In this method, the domain is discretized by a set of mass carrying particles which automatically guarantees the mass conservation. However, this leads to an additional drawback which is that the quality of the discretization can not be easily adapted according with the evolution of a problem since the addition or removal of particles would produce a change in the considered material density [1,3,23].

One of the first reported scientific works using SPH for the simulation of heat transfer and mass was done by Cleary [24]. Later on an improved version of SPH was presented by Cleary and Monaghan for heat transfer numerical simulation with a discontinuous and highly temperature dependent conductivity [25]. Cleary proposed the use of SPH for fluid flow simulation coupled with heat transfer and solidification in casting [26]. Recently, Cao et al. reported an implementation of SPH to simulate coupled fluid flow and heat transfer for mould filling process of a disc [27]. Ren et al. presented an improved particle method based on SPH for non-isothermal viscoelastic fluid simulation in mould filling process [28].

A Lagrangian truly meshless approach that can overcome some of the problems in SPH formulation and in other strong-form meshfree methods, especially those related to the treatment of the boundary conditions is FPM [23]. This method has been developed by Kuhnert in Ref. [29] at the Fraunhofer-Institut für Techno-und Wirtschaftsmathematik, in Kaiserslautern, Germany. FPM has shown to be far superior to traditional mesh-based methods and some other meshfree method for problems involving rapidly changing flow domains with respect to time, multiphase or free surface flows [30–34], and radiative heat transfer problems [35]. Similarly to SPH, FPM uses a set of finite nodes scattered within a problem domain as well as on its boundaries, which do not carry mass. This provides the flexibility to add or remove nodes wherever and whenever needed and it lets to easily develop adaptive schemes to locally modify the discretizing point cloud during a simulation. All these facts make FPM especially suitable for practical problems involving fluid dynamics with rapidly changing domains. Therefore, in this work the application of the FPM of Kuhnert to free surface thermal flows in the context of mould filling processes is proposed being the first time, to the authors knowledge, that this approach is applied in order to solve practical engineering industrial processes in particular with potential application in the area of metal casting. The FPM practical implementation to thermal flows in this context, the simplicity for the implementation and treatment of boundary conditions on complex problems as well as the extension of the range of applications for FPM is the motivation behind this research work.

The structure of the paper is as follows: section 2 introduces the governing equations, section 3 shortly describe the numerical scheme for solving the system of PDEs, section 4 presents the basic ideas behind FPM followed by some issues regarding the numerical implementation of FPM in section 5. The numerical results are reported in section 6 and finally some conclusions are given in last section.

2. Governing equations

The governing equations during mould filling in metal casting, considering that molten metal is incompressible, are the incompressible Navier-Stokes equations in a laminar regime coupled with the convective heat transfer equation. It is well known that during the cooling process within the mould the molten metal experiments a transition from Newtonian to Non-Newtonian fluid in the range between liquidus and solidus temperatures in which solidification occurs. There are several studies on the rheological behaviour of different metal alloys which indicate huge discrepancies between different studies. Most of the studies show viscosities that differ in several order magnitudes, however, they agree on the fact that the viscosity follows a power-law relationship for which different parameters, coefficients and reference values are defined [36,37]. Moreover, there is no detailed reference results in the scientific literature about cooling or solidification processes during mould filling in metal casting to compare with. By these