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Heat flux and temperature determination in a cylindrical element with the use of Finite Volume Finite Element Method

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ABSTRACT

This paper presents a method to determine the local heat flux and temperature distribution on a thick cylindrical surface through the solution of an inverse heat conduction problem (IHCP). In particular, the method is developed for cases when traditional measurements cannot be performed due to difficulties in accessing the surface; a common situation in engineering and research practice. In particular, inverse methods are sensitive to measurement errors, for example, those caused by a mispositioned temperature sensor on a solid body. Hence, this paper proposes a method that is insensitive to, and can accurately be used, even in presence of these errors. The proposed method lowers the error in the local heat flux and temperature determination hence making the inverse methods more reliable.

The proposed method is based on the finite element-finite volume method (FEMFVM) approach. It is tested using measurement data obtained on a laboratory stand. The measurements are performed using thermocouples installed inside a solid cylindrical wall.

The method is tested and compared to other numerical methods, including the finite volume method (FVM) and the finite element method (FEM). Extensive additional calculations are carried out to evaluate the impact of random measurement errors on the accuracy of the proposed method.

The calculation results prove that the new method is suitable and useful in the case of complex geometrical shapes and enables accurate determination of parameters such as the heat flux, and the outer surface temperature.

1. Introduction

The problem of determining heat flux occurs in different areas of technology and engineering applications such as combustion chambers, industrial boilers, metallurgy, castings, buildings thermal management, aerospace engineering and others [1–4]. However, it is not always possible to access a surface where the heat flux or heat transfer coefficient is determined [5,6]. Hence, one needs to measure the solid body temperature at discrete internal points to deteremine the heat flux through the use of inverse heat conduction equations. The inverse methods can be extremely useful when flow over a surface has to remain undisturbed restricting the access for direct measurements. Such inverse method is harder to perform (than direct measurements) and is limited, for instance, by also random measurement errors and material properties uncertainties. However, the advantage outweighs the disatvantages in some cases and leaves inverse methods as widely used [6,7]. Extensive research had been performed to develop the methods and instruments for measuring transient heat flux variation at the surface of the solids. A simple device that uses one temperature sensor is presented in Ref. [5]. The method of heat flux determination utilizes Inverse Heat Condition Problem (IHCP) based on the Levenberg-Marquardt (L-M) algorithm. The device was used to study the effect of thermocouple location, signal length, and sampling frequency on the estimated heat flux values. The knowledge of heat flux distribution is, in particular, required when the problem under investigation is a bolier furnace. For example, the heat load on the combustion chamber height can be determined as shown in Refs. [8,9]. Because of high temperature in combustion chambers, heat flux density measurements require special designs and techniques. Zhang et al. [10] presented a heat flux meter based on two thermocouples (TCs) measuring solid metal temperature embedded in the boiler walls between the adjacent tubes. The front side measures the heat flux in a boiler combustion chamber while the rear is cooled down by a water-cooled system [10,11]. The heat flux meter utilizes 1D heat conduction phenomenon (while the heat flux meter presented in this paper takes into account 2D heat conduction effects). Taler et al. [12] developed transient heat flux meter where TC

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Nomenclature

A	temperature coefficient
c	specific heat J/(kgK)
cs	control surface
k	thermal conductivity, W/(m K)
M	number of time steps
MAPRE	mean absolute percentage relative error, %
N	shape function
q	heat flux W/m^2
r	radial coordinatem
RMSE	root mean squared error, °C
S	area of control surface m^2
T	temperature K
x	x-coordinate
y	y-coordinate

Vectors and matrices

$\mathbf{J}$	Jacobian matrix
$\mathbf{n}$	normal vector
$\mathbf{q}$	Heat Flux vector

Greek symbols

$\Delta\tau$	time step, s
Δr, $\Delta\varphi$	coordinate increment m
Ω	area of control volume m^2
η	local coordinate in vertical direction
ξ	local coordinate in horizontal direction
ρ	density kg/m^3
σ	standart deviation
τ	time s
φ	angular coordinate

Subscripts

B	boundary (inner surface)
D	direct region
I	inverse region
U	upper surface
i	subcontrol volume number
j	boundary node number
max	maximum RMSE
mean	mean RMSE

Superscripts

*	distorted coordinate increment
N	time step number

is emerged in water wall tube solid body. The device proposed by the authors determines heat flux from the temperature measurements where the four thermocouples are located at the fireside part of the tube and one is attached to the unheated rear surface of the tube. Sankar et al. [13] present recent review of boiler heat flux meters.

Heat flux determination finds it application in the field of materials processing [14–16] where special inverse methods are adopted in heat transfer identification in the casting. Udayraj et al. [14] propose a method that uses a 2D IHCP algorithm based on conjugate gradient method focusing on reducing the number of sensors required for the mold surface heat flux determination. It was found that 5 TCs meets the compromise between accuracy and the lowest TCs number.

An inverse analysis of instantaneous heat fluxes from solidifying steel to the surface of twin roll casters and aluminum to plasma-coated metal substrates was carried out in Ref. [15]. Circumferential and axial heat conduction was assumed to be negligible compared to the radial heat conduction for the range of the roll speeds used in the experiment. The least squares method in conjunction with a second regularization method was used to solve a 1D IHCP in the cylindrical coordinate system. The Gauss-Newton method was applied to determine the surface heat flux based on the time responses of a pair of thermocouples located at two different positions inside the rolls. Unnikrishnakutup et al. [17] studied a gas tungsten arc welding process to estimate heat flux influence on the process efficiency and Gaussian heat distribution radius. The unknown heat flux value was determined following the solution of IHCP while Levenberg-Marquardt method was used as the regularization of the inverse problem. It was pointed out that a small inaccuracy on the TC position caused errors in heat flux evaluation.

Yang and Chen [18] solved an inverse heat conduction problem encountered in a disc brake system. The generated heat and the temperature field in the disc are identified from the knowledge of temperature measurements taken inside the disc. To estimate the unknown space- and time-dependent heat flux on the disc surface the least squares method was used. The conjugate gradient method used in Ref. [18] transforms the IHCP into three problems, namely, the direct, the sensitivity and the adjoint problem.

Sousa et al. [19] performed measurement and calculation to estimate heat flux at the tool-piece interface during the drilling process. They developed an inverse technique based on the Green's function and dynamics observers. A 3D transient model with moving interface was employed. Since direct temperature measurements at the tool-workpiece interface are very complex the temperature history from the thermocouple attached to the workpiece surface was used as the input data for the inverse problem. Assessments of the inverse procedure was performed by the comparison of the calculated and measured temperature field. Wang et al. [20] analyzed thermal issues in bone grinding processes using inverse methods to calculate the heat flux incorporating sequential function specification method and sequential quadratic programming. Numerical and experimental data were used and cross-validated the method.

A general space marching method for solving the transient multidimensional inverse heat conduction problem (IHCP) was proposed by Taler and Zima [21]. To reduce the sensitivity of the inverse method to random errors the measured temperature histories were smoothed by moving average filter based on the Gram's polynomials approximating nine successive measurement points. Taler et al. [22,23] developed a space marching method for determining the transient temperature and stress distributions in horizontal pressure components. The temperature difference over the circumference of horizontal pressure vessels was taken into account. Temperature and thermal stress distributions were determined indirectly by measured temperature values at selected points on the outer surface of a pressure element. The determined transient temperature distribution allowed the authors to calculate the thermal stresses using the Finite Element Method. Measured pressure changes are used to derive pressure-caused stresses. The calculated temperature histories were compared with the experimental data at selected interior points. The presented method of thermal stress control was applied in various power plants in Poland. Similar problems were analyzed by Lu et al. [24,25] who studied thermal stratification in flow through elbows used for mixing hot and cold fluids. The movement of the interface between the hot and cold fluids over time causes fluctuations of the inner wall temperature which can lead to the variation of thermal stresses in the pipeline wall and, hence, thermal fatigue of the elbow. The objective of the inverse problem was to estimate the