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Analytical solution for the soil freezing process induced by an infinite line sink

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ABSTRACT

One-dimensional soil freezing process induced by an infinite line sink is investigated, a problem that is generally encountered during the application of the artificial ground freezing technique. The special feature of the freezing process of soil is that the amount of liquid water in the soil pores decreases gradually with the soil temperature decreasing, and thus latent heat is continuously released. After approximating the continuous phase change process of the soil with a multi-step phase change process of a polymorphous material, the original problem is transformed to a cylindrical multiphase Stefan problem. Using the similarity transformation technique, an analytical solution for the problem is developed, in which there are coefficients that need to be determined by a group of non-linear equations. A theoretical proof is presented showing that these coefficients can be appropriately determined by the non-linear equations. The computational results of the analytical solution are compared with those of the numerical solution, and the two solutions are in good agreement. The effects of the variation of different parameters on the movement of freezing front are studied. The results show that neglecting the existence of unfrozen water will underestimate the freezing front velocity. The relative error of this underestimation under different conditions is also investigated and discussed. In situations that the unfrozen water content is given as a discrete function of the temperature, the analytical solution can be applied directly, and an illustrative example is presented.

1. Introduction

There are many processes in engineering or physics that can be modelled as heat conduction with a line source or sink. For example, during the application of a ground-coupled heat pump system [1–3], the length of the heat exchanger buried in the ground is much larger than the radius of the exchanger, thus the heat exchanger is usually simplified as a line, and the physical process between the heat exchanger and the surrounding soil is modelled as heat conduction induced by a line source or sink.

The potential applications in both engineering and physics stimulate the theoretical research on the line source model (heat conduction process with a line source). Great efforts have been spent and many analytical solutions for different types of line source models have been developed. The most-widely-used model is Kelvin's line source model, the solution of which can be found in classical text books [4,5]. Kelvin's model considers the radial heat conduction process caused by an infinite line source (the length of the line is infinite), and the thermal properties of the material around the line source are constant. Zhou et al. [6] made some extension of Kelvin's model by including temperature-dependent thermal properties; they transformed the problem to a multi-phase Stefan problem with no latent heat at the moving boundaries, and then developed an analytical solution using the similarity transformation technique. Li et al. [7,8] studied the infinite line source problem in a composite region and developed an analytical solution; the composite region they considered is composed of the grouting material in the borehole and the soil surrounding the borehole. There are also finite line source models (the length of the line is finite) that include the heat conduction in the vertical direction, and these models are two or three dimensional. Eskilson [9], Zeng et al. [10] and Diao et al. [11] investigated the vertically buried finite line source in a semi-infinite region with constant surface temperature, and they presented an integral form solution. Lamache et al. [12] rearranged the integral form solution, and improved the efficiency for the numerical calculation of the solution. During the application of ground-coupled heat pump systems, the borehole in which the heat exchanger is installed may be inclined. Taking this into account, Cui et al. [13] presented a finite inclined line source model and developed a

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corresponding analytical solution. In situations when there is the presence of groundwater, the temperature field around the line source may be affected by the flow of groundwater. Molina-Giraldo et al. [14] included the effect of groundwater movement, and developed a moving finite line source model using heat conduction theory in moving solids. Most of the solutions for two or three dimensional models are developed under the first type of boundary condition; however, the third type of boundary condition at the ground surface is more realistic. In order to study the near surface effects on borehole heat exchangers, Rivera et al. [15] established an analytical solution for a moving finite line source model under the third type of boundary condition. Layered soils are also commonly seen during the application of the ground-coupled heat pump system, therefore Abdelaziz et al. [16], Zhou et al. [17] developed analytical solutions for a finite line source model in layered soils.

Although many analytical solutions for different types of line source models have been developed, the phase change of the soil is not considered in these solutions. The phase change of the soil is inevitable for some applications such as the engineering in cold regions, and most of the solutions developed for the soil freezing (thawing) process are numerical solutions [18,19]. For the line source model taking into account the phase change of the soil, which is often encountered during the application of the artificial ground freezing (AFG) technique [20,21], there is no analytical solution reported.

The AGF technique is widely used in underground engineering, especially in situations of weak and soft soils. Taking the shaft construction process for example, freezing pipes are buried vertically in the ground as shown in Fig. 1. The refrigerant circulates in the freezing pipes and causes the surrounding soil to freeze. When the thickness of the frozen soil reaches certain level, the strength of the frozen wall will be high enough to bear the pressure from the soil outside the frozen wall, and then the excavation of the soil and the construction of the shaft lining may proceed.

The strength of the frozen wall is mainly determined by its thickness and average temperature, thus temperature prediction is important for the application of the AGF technique. The temperature variation during the application of the AGF technique greatly depends on the configuration of the freezing pipes. However, at the initial stage of the freezing process, the temperature around a single freezing pipe can be analyzed independently, and the effects from neighboring freezing pipes are usually neglected.

This paper investigates the variation of the temperature field around a single freezing pipe, in which the freezing pipe can be simplified as a line since its length is much larger than its radius. Section 2 presents the governing equations for the soil freezing process induced by a heat line sink and transforms the problem to a multiphase Stefan problem in cylindrical coordinates. Section 3 establishes the analytical solution using the similarity transformation technique, while section 4 proves that the coefficients in the solution can be uniquely determined by a group of non-linear equations. Section 5 presents computational examples of the solution, followed by Section 6, with some conclusions.

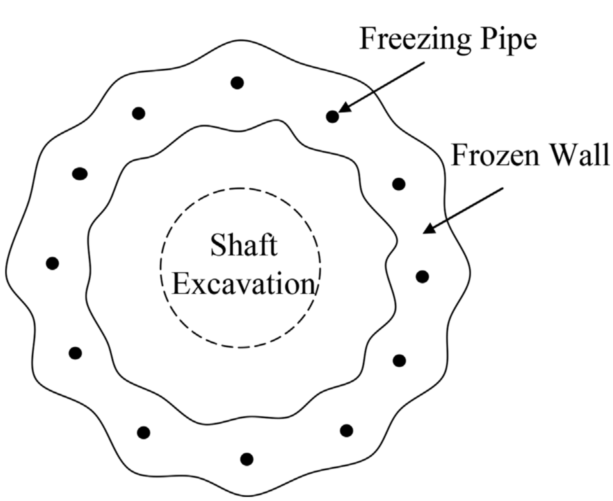

Fig. 1. A plan for the application of the AGF technique in shaft engineering.

2. Mathematical description of the problem

2.1. Governing equations for the original problem

The so-called soil freezing process is in fact the phase change of the liquid water to ice in the soil pores. Due to the small radius of the soil pores and interfacial forces, the liquid water in the soil pores does not freeze completely when the temperature reaches the soil freezing point, and there is always unfrozen water in the soil pores below freezing temperatures [22]. With the soil temperature decreasing, the unfrozen water content in the soil pores also decreases. The relation between the unfrozen water content and the temperature is usually called the soil freezing characteristic, and can be described by Ref. [23].

$$\theta_u = \frac{\rho_d}{100\rho_w}\exp(0.2618 + 0.5519 lnS - 1.4495 S^{-0.2640} ln|T|) \tag{1}$$

in which θ_u is the volumetric unfrozen water content, ρ_d is the soil dry density (kg/m^3), ρ_w is the water density (kg/m^3), S is the specific surface of the soil particles (m^{-1}), and T is the soil temperature (°C).

In this paper, we consider the soil freezing process induced by an infinite line sink. Since the infinite line model is involved, heat conduction in the direction along the line sink is neglected, and only radial heat conduction process is considered. The initial temperature of the soil around the line sink is V (°C) uniformly, and the magnitude of the line sink is Q (W/m).

The governing equations for the problem can be written as

$$C_v\frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) - L_f\rho_w\frac{\partial \theta_u}{\partial t}, \quad r > 0 \tag{2}$$

$$\lim_{r\to 0}\left(2\pi rk\frac{\partial T}{\partial r}\right) = Q \tag{3}$$

$$T(r, 0) = V, \quad r > 0 \tag{4}$$

in which r is the cylindrical coordinate (m), and t is the time (s); C_v and k are the volumetric heat capacity (J/(m^3·K)) and the thermal conductivity (W/(m·K)) of the soil, and they both are functions of θ_u; L_f is the latent heat of water (334 kJ/kg). In the frozen zone, the relation between θ_u and T is described by Eq. (1), while in the unfrozen zone, θ_u equals the initial water content θ_0.

2.2. Governing equations after problem transformation

The objective of this paper is to develop an analytical solution for the soil freezing process induced by an infinite line sink. However, it is impossible to solve the governing equations (2)–(4) directly with the soil freezing characteristic described by Eq. (1), and some simplifications should be made before an analytical solution can be established.

The main difficulty for obtaining an analytical solution of the original problem is the continuous phase change process of the soil. Fig. 2 shows a schematic diagram for the continuous curve of the soil freezing characteristic based on Eq. (1). The first step before the construction of the solution is to approximate the continuous curve of the soil freezing characteristic with a step-type curve [24], as shown in the figure; from another perspective, this process is essentially approximating the material of soil with a polymorphous material.

Assuming that the polymorphous material has $N+1$ phases, there are also different ways to choose the step-type curve for this material. Denoting the phase change temperatures of this polymorphous material in ascending order as T_{pc1}, T_{pc2}, …, $T_{pc(N-1)}$, $T_{pcN} = T_f$, with T_f the soil