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Stability, convergence and optimization of interface treatments in weak and strong thermal fluid-structure interaction

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ABSTRACT

This paper presents the stability, convergence and optimization characteristics of interface treatments for steady conjugate heat transfer problems. The Dirichlet-Robin and Neumann-Robin procedures are presented in detail and compared on the basis of the Godunov-Ryabenkii normal mode analysis theory applied to a canonical aero-thermal coupling prototype. Two fundamental parameters are introduced, a "numerical" Biot number that controls the stability process and an optimal coupling coefficient that ensures unconditional stability. This coefficient is derived from a transition of the amplification factor. A comparative study of these two treatments is made in order to implement numerical schemes based on adaptive and local coupling coefficients, with no arbitrary relaxation parameters, and with no assumptions on the temporal advancement of the fluid domain. The coupled numerical test case illustrates that the optimal Dirichlet-Robin interface conditions provide effective and oscillation-free solutions for low and moderate fluid-structure interactions. Moreover, the computation time is slightly shorter than the time required for a CFD computation only. However, for higher fluid-structure interactions, a Neumann interface condition on the fluid side presents good numerical properties so that no relaxation coefficients are required.

1. Introduction

Conjugate heat transfer (CHT) analysis is a simulation process that addresses the thermal interaction between a body and a fluid flowing over or through it. Conjugate heat transfer problems occur whenever fluid convection and solid material conduction are taken into account simultaneously. The concept of "conjugated problems" was first formulated in the early 1960s by Perelman [1]. As a result, heat transfer has been often investigated as a coupled problem [2] since this mutual interaction has become increasingly important in many numerical simulations.

CHT analysis can be performed in a monolithic manner in which the equations are solved simultaneously in a single solver [3,4] but such an approach is not flexible and cannot be pursued with commercial codes. In contrast, partitioned techniques allow the direct use of a specialized solver for each subdomain, offering significant benefits in terms of efficiency and code reuse. In this strategy, the solution is advanced in time separately within each partition [5,6,7].

However, the time lag due to the sequential treatment in partitioned procedures can have a detrimental effect on the stability and performance leading to slow convergence. In a fluid-structure interaction (FSI), this staggered process generally leads to spurious energy production. Specific numerical treatments are proposed in the literature to overcome these difficulties. Examples include a combined interface boundary condition, proposed by Jaiman et al. [8,9], an interface correction controlled by a coupling parameter [10] and the use of a specific partitioned algorithm in conjunction with a relevant Robin condition [11].

In CHT, we experience the same problems and constraints. There are many similarities between FSI and CHT. A variety of approaches have been employed based on finite elements, finite volumes, boundary elements and spectral approximations [12-17]. Numerical methods are also required to counteract the intrinsic destabilizing effect of the time lag and time discrepancy between each sub-domain. It is likewise standard to enforce continuity at the interface between the fluid and the solid. This can be achieved by using one or two coupling coefficients that control stability. Many papers in CHT have sought to improve the interface conditions by adopting simple model problems from which the interface conditions and coupling coefficients can be derived.

In many cases, the model problems show that the structure of complex multiphysics systems is often as important as the behavior of the individual components themselves. Indeed, fluid and solid domains

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Nomenclature

a	thermal diffusivity [$m^2.s^{-1}$]
Bi	Biot number
$Bi^{(\Delta)}$	mesh Biot number
Bi_ν	numerical Biot number
D	Fourier number
$\overline{D}$	normalized Fourier number
F	inviscid and viscous flux
g	temporal amplification factor
h	heat transfer coefficient [$W.m^{-2}.K^{-1}$]
n	coupling iteration
ε	error tolerance
λ	thermal conductivity [$W.m^{-1}.K^{-1}$]
K	thermal conductance [$W.m^{-2}.K^{-1}$]
N	number of cells at interface cells
q	heat flux [$W.m^{-2}$]
t	time [s]
T	temperature [K]
Γ	thermal conductivity matrix
w	fluid conservative quantity
$y+$	non-dimensional wall distance
z	complex variable
α	coupling coefficient [$W.m^{-2}.K^{-1}$]
κ	spatial amplification factor
ν	FVM/FEM parameter
ρ	density [$Kg.m^{-3}$]
Λ	characteristic size [m]
Ω	domain/partition
Δy	size 1st cell [m]
Δt	time step [s]
ν_f	inward unit normal to the fluid domain
ν_s	inward unit normal to the solid domain

Subscripts

c	coupled
f	fluid domain
s	solid domain
ref	reference value
ν	numerical

Superscripts

n	temporal index
min	minimum
max	maximum
opt	optimal
$(\dot{\cdot})$	unknown value

can interact in many different ways. Model problems are a means to understand and quantify these dynamic interactions. For instance, in FSI, the added-mass was highlighted by Causin et al. [18] from a simplified model problem. Using a different model problem and a normal mode analysis, Banks and Sjögreen [19] obtained a similar result. Similarly, in CHT, the nature of the instabilities derived from a simplified 1D model can provide insights into the potential instabilities in the computation of 2D/3D flows.

The behavior of the interface conditions in CHT is also often studied using a normal mode analysis. For instance, the pioneering work of Giles [20], the new procedure applied to CHT proposed by Roe et al. [21], the composite grid solver introduced by Henshaw and Chand [22] and the stability analysis in transient CHT presented by Kazemi-Kamyab et al. [23]. An interesting alternative is the steady-state approach described by Verstraete and Scholl [24]. It should be mentioned that there are other methods of investigation, such as the energy method, to analyze well-posedness and stability [25].

By using a thermal model problem, Errera and Chemin [26] have identified a numerical transition that can be expressed mathematically. This fundamental result has been derived from a normal mode stability analysis based on the theory of Godunov-Ryabenkii [27,28,29]. This transition results in an optimal coefficient in terms of stability and convergence.

The formulation of the optimal coefficient was published relatively recently. Consequently few CHT computations have been reported, that take advantage of it. However, some interesting results have been reported. For instance, in a steady CHT computation of an effusion cooling system [30], the CPU time necessary to converge was divided by a factor of ten, in contrast to a conventional method. In another work devoted to testing systematically the values of various coupling coefficients [31], it was shown that the optimal coefficient in combination with a Dirichlet-Robin procedure (temperature prescribed to the fluid sub-domain) could be applied in an efficient manner as a tool for predicting and obtaining excellent stability properties. This result was confirmed recently in a complex set-up of a heating cell found in various industrial applications (conveyors, reheat furnaces). A systematic comparison of various coefficients was undertaken and it was shown that the optimal coefficient outperformed the previous results in the literature [32]. In transient CHT problems, optimal coefficients can also be applied to analyze heat transfer during a full transient flight cycle as shown in Ref. [33] where specific numerical characteristics at the interface were provided.

Previous studies suggest that the one-dimensional normal mode analysis could provide relevant coefficients directly applicable to industrial CHT problems. These promising results have been obtained by using Dirichlet-Robin conditions, a method widely used in the literature. However, ideally, Robin conditions on either side of the interface should be considered because they introduce local simplified models whether for FSI [18,34] or CHT [35]. Yet, this general Robin-Robin interface condition results in a very large family of schemes and we prefer, as a first step, to consider the two conditions that form the basis of this general approach. Thus, the present paper is confined to two complementary interface treatments:

- A Dirichlet-Robin procedure: the temperature obtained from the solid is applied on the fluid side, and a "relaxed heat flux" is in turn used as a boundary condition for the solid.
- A Neumann-Robin procedure: the heat flux obtained from the solid is applied on the fluid side and a Robin condition is in turn used as a boundary condition for the solid.

The above CHT interface procedures are the most commonly used conditions in the literature. The goal of this paper is to present them in detail and to provide, for the first time, their remarkable properties, in particular the temporal and spatial amplification factor, instability zones, upper and lower stability bounds and optimal coefficients on the basis of a canonical coupling prototype. These results will be summarized in tables where the numerical properties are evaluated according to the nature of the fluid-solid interaction. Moreover, a comparative study of these two treatments will be made in order to implement efficient numerical schemes, that is to say schemes based on adaptive and local coupling coefficients, with no arbitrary relaxation parameters, and with no assumptions on the temporal progression of the fluid domain.

The paper is composed as follows. The theoretical study is presented first (Section 2) and the precise conditions to obtain optimal coefficients